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**Exercise 5-1:** Let the convolution integral be defined as

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau. \tag{1}$$

Show the following properties of the convolution integral and Laplace transforms:

(a) $f * g = g * f$

(b) $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$. For the Laplace transform, you can change the limits of integration from $(-\infty, \infty)$ to $[0, \infty)$. Also, don't worry if the expressions are equal up to a constant.

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**Exercise 5-2:** Use the Laplace transform to solve the following ODEs:

(a) $\ddot{x} + 5\dot{x} + 6x = u(t)$

(b) $\ddot{x} - 2\dot{x} + 2x = u(t)$

Solve each of these for the following initial conditions and forcing functions (hint: use the Laplace transforms from above so simplify the expression in the frequency domain. I would not recommend using convolution, if you can avoid it):

(i) Step response: A step input (i.e., $u(t)$ is a Heaviside function) with zero initial conditions

(ii) Impulse response: An impulsive input (i.e., $u(t)$ is a Delta function) with zero initial conditions

For each case, plot your solution and also plot Matlab's solution using the `step` and `impulse` commands.

(iii) Initial condition response: $u(t) = 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$.

(iv) Initial condition response: $u(t) = 0$ with initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$.

Summary: Solve both equations (a) and (b) using the forcing and initial conditions from (i)-(iv).

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**Exercise 5-3: [ For Fun ]:** The convolution integral and the impulse response may be used to simulate how an audio signal would sound under various conditions, such as in a long hallway, in a concert hall, or in a swimming pool.

The basic idea is that you can record the audio response to an impulsive sound in a given location, like a concert hall. For example, imagine that you put a microphone in the most expensive seats in the hall and then record the sound from a shotgun blast up on the stage (do not try this!!). Then, if you have a “flat” studio recording of some other audio, you can simulate how it would have sounded in the concert hall by convolving the two signals.

Please download and unzip sounds.zip to find various sounds and impulse response filters.

Using the `conv` command, convolve the various audio files (labeled sound1.wav, ...) with the various filters (labeled FilterXYZ.wav, ...). To load these audio files, use the `wavread` command. It is best to add 10% of the filtered audio (aka “wet” audio) to 90% of the original audio (aka “dry” audio). Listen to the filtered audio (as well as the original audio and the impulse response filters) using the `>>sound(signal,11025);` command (note that each sound has a sampling rate of `FS=11,025`). However, you will need to be careful when adding the 10% filtered and 90% unfiltered signals, since the filtered audio will not necessarily have the same length as the filtered audio.

Please include your code in your homework submission.

There is a great video explaining how to actually create these impulse responses:

http://www.audioease.com/Pages/Altiverb/sampling.php

He glosses over the mathematical details, but you can get a good idea of how it works.