Exercise 2-1 Solve the one-dimensional Laplace equation for the equilibrium temperature distribution in a finite rod for the following boundary conditions:

1. $u_x(0) = 0$ and $u(L) = B$,
2. $u(0) = B$ and $u_x(L) = -a$,
3. $u(0) + u_x(0) = 0$ and $u(L) = B$.

Please also draw a quick sketch for each boundary condition. If you need an initial condition for any reason, you may assume that $u(x, 0) = f(x)$.

Exercise 2-2 Solve the modified Laplace equation with a source term (Poisson's equation!):

$$u_{xx} = -Q(x), \tag{1}$$

for $Q = x^2$. Use fixed temperature boundary conditions so that $u(0) = 0$ and $u(L) = 0$.

What can you say about the heat flux at $x = 0$ and $x = L$?

Plot the temperature distribution for $L = 10$.

Exercise 2-3 Please solve for the equilibrium temperature distribution of the following heat equation, $u_t = u_{xx} + 2$ (i.e. with $Q = 2$), with the following initial and boundary conditions: $u(x, 0) = f(x)$, $u_x(0, t) = a$, and $u_x(L, t) = b$. Does this distribution exist for all values b? If not, then what are the values of b that have solutions?

Exercise 2-4 Derive the 3D Laplacian operator in cylindrical coordinates (R, θ, z), starting from the Laplacian in Cartesian coordinates (x, y, z).