ME565 - Winter 2018 **Due: 5:00:00pm Tuesday January 16^{th}, 2018**
Homework 1

Exercise 1-1 Express the following in the form $a + bi$ (for real a and b) and also in the form $Re^{i\theta}$ (for real R and θ):

(a) $\frac{1}{2-4i}$

(b) $\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{1}}{2}i\right)^4$

(c) $(-i)^2, (-i)^3, (-i)^4, (-i)^5, \ldots$

Exercise 1-2 Find all solutions of

(a) $e^z = -1$

(b) $e^z = -i$

(c) $e^z = 1 + i$

Exercise 1-3 Find all solutions of

(a) $z^2 = -i$

(b) $z^3 = -i$

(c) $z^6 = 1$

(d) $z^3 = -1 - i$

Exercise 1-4 Find all analytic functions $f = u + iv$ with $u(x, y) = 2xy$. Simplify the expression $f(z)$ as much as possible.

Exercise 1-5 Evaluate the following integral

$$\int_C f(z)dz$$

where $f(z) = 1/z$ and C is parameterized by $z(\theta) = \sin(\theta) + i\cos(\theta)$ for $0 \le \theta \le 2\pi$.

Exercise 1-6 Verify the following functions $f(z)$ are analytic for all $z = x + iy$. Use the Cauchy-Riemann conditions (Hint: find a way to express these functions as $f(z) = u(x, y) + iv(x, y)$).

(a) $f(z) = e^z$

(b) $f(z) = \cos^2(z)$

Exercise 1-7 In Matlab, write a script to evaluate the following integral:

$$\int_C \frac{f(z)}{z} dz$$

for the curve C parameterized by $z = Re^{i\theta}$ for $R = 1$ and $0 \leq \theta \leq 2\pi$. (Note that $dz = iRe^{i\theta}d\theta$).

Verify the Cauchy integral formula $\left(i.e. \ \int_C \frac{f(z)dz}{z-a} = 2\pi i f(z)\right)$ for $a = 0$ with

(a) $f(z) = e^z$

(b) $f(z) = \cos^2(z)$

(c) Now, shift the curve C so that it is still a circle with radius $R = 1$, but now centered at $-2 + 2i$. Does the integral in part (a) change?

(d) Are your answers consistent with Exercise 1-6?