Mechanical Engineering 565
Midterm Exam
Winter Quarter 2018

Question	Score
1	/ 10
2	/ 10
3	/ 10
4	/ 10
5	/ 30
6	/ 30
Total	/100

Name:

UW NetID:

UWStudent ID:

Instructions: This is a 4 hour, take-home exam, and everybody is expected to abide by the honor code and follow the rules below. The 4 hours you use for the exam do not need to be consecutive. For example, you can work on the exam for 2 hours on Wednesday and another 2 hours on Thursday. Please be reasonable, and try not to split the exam up into more than four blocks of time.

You are allowed to use the course notes (online pdfs and your own handwritten notes), online ME565 lecture videos, and *your own* homework solutions on the exam. All other resources are prohibited, including: the internet, other books, discussing the exam with other people.

Bona Fortuna!!

Problem 1: Find all solutions of

(a) $e^z = -1$

(b) $e^z = 1 - i$

(c) $z^3 = i$

(d) $z^4 = -1$

(e) $(\sqrt{2} + \sqrt{2}i)^4$

Please express in the form $a + ib$.

Problem 2: This problem concerns analytic functions of a complex variable $z = x + iy$.

(a) Are there any analytic functions $f = u + iv$ with $u(x, y) = x^2 + y^2$? If so, give an example, and if not, explain why not.

(b) Consider the function $f(x, y) = u(y) + iv(x)$ so that u only depends on y and v only depends on x. What are the possible functions f of this form that are analytic everywhere; please simplify f as much as possible.

Problem 3: Please compute the following integrals of the following function

$$I = \int_C \frac{z^2}{(z^2+1)(z^2+4)} dz$$

for the following contours:

(a) C is a circle of radius 0.5 centered at $-2i$.

(b) C is a circle of radius 0.5 centered at i.

(c) C is a circle of radius 0.5 centered at 1.

Be sure to justify your reasoning.

Problem 4: Solve the one-dimensional Laplace equation for the equilibrium temperature distribution in a finite rod of length L for the following boundary conditions:

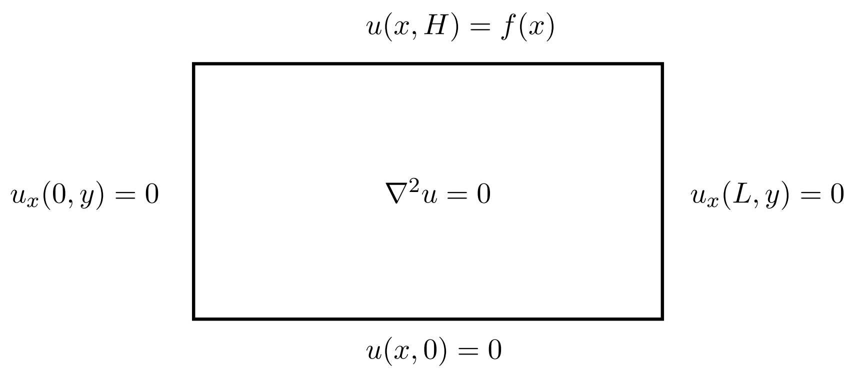
1. $u(0) = 1$ and $u(L) = 2$
2. $u(0) + u_x(0) = A$ and $u(L) + u_x(L) = B$

Please also draw a sketch for each boundary condition.

Problem 5: Solve for the equilibrium temperature distribution using the 2D Laplace equation on an $L \times H$ sized rectangular domain with the following boundary conditions:

1. Left: $u_x(0, y) = 0$ (insulating)
2. Bottom $u(x, 0) = 0$ (fixed temperature)
3. Top: $u(x, H) = f(x)$ (zero temperature)
4. Right: $u_x(L, y) = 0$ (insulating)

Solve for a general boundary temperature $f(x)$. Also solve for the particular temperature distribution $f(x) = \sin(4\pi x/L)$.



How would this change if the left and right boundaries were fixed at zero temperature? (you don't have to solve this new problem, just explain in words what would change)

Problem 6: Heat conduction in a thin circular ring is described by the following PDE

$$u_t = u_{xx}, \text{ for } t \geq 0. \tag{1}$$

with periodic boundary conditions in space ($x \in [0, 2\pi]$):

$$u(0,t) = u(2\pi, t), u_x(0,t) = u_x(2\pi, t), \text{ etc.} \tag{2}$$

Solve using the separation of variables, i.e. $u(x,t) = X(x)T(t)$. Find a general solution (i.e. for arbitrary initial condition $u(x,0) = f(x)$) and describe the asymptotic behavior of *all* solutions as $t \to \infty$. Provide a physical interpretation of this behavior.