ME565 - Winter 2018 **Due: 11:59pm Sunday March 11^{th}, 2018**
Homework 6

Exercise 6-1: Please solve the following lightning-round complex analysis problems:

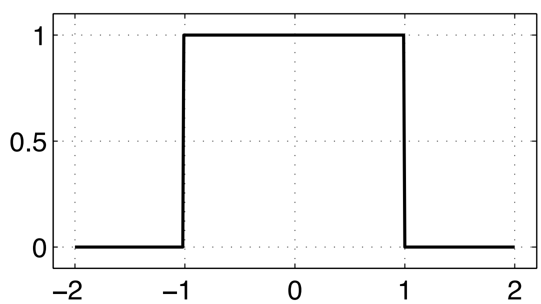
- Find all solutions to $e^z = -i$,
- Find all solutions to $z^3 = -8i$,
- What is the solution to the following integral:

$$\int_C \frac{z^3}{z-5} dz$$

For C a circle of radius 1 centered about the origin? How about for C a circle of radius 10 centered about the origin?

Exercise 6-2: Compute, by hand, the **Fourier series** representation for the following square wave defined on $x \in [-2, 2]$:

$$f(x) = \begin{cases} 0 & -2 < x < -1 \\ 1 & -1 \leq x \leq 1 \\ 0 & 1 < x < 2 \end{cases}$$



In Matlab, plot the mode coefficients a_n and b_n for the first 100 cosine and sine modes (i.e. for the first $n = 1$ to $n = 100$).

Also, plot the approximation using $n = 10$ modes on top of the true square wave. Try $\Delta x = 0.01$.

Next, compute the **Fourier transform** by hand for this function by extending the domain to infinity.

Exercise 6-3: Consider the following wave equation initial value problem:

$$u_{tt} = u_{xx}$$

on the semi-infinite domain $0 \leq x < \infty$ with the following initial and boundary conditions:

$$u(x,0) = \sin(4x), u_t(x,0) = 0$$
$$u(0,t) = te^{-t}.$$

Please solve for $u(x,t)$.

Exercise 6-4: Consider the following PDE for a vibrating string of finite length L:

$$u_{tt} = c^2 u_{xx}, \quad 0 \leq x \leq L$$

with the following initial conditions

$$u(x,0) = 0, \qquad\qquad u_t(x,0) = 0;$$

and boundary conditions

$$u(0,t) = 0, \qquad\qquad u_x(L,t) = f(t).$$

Solve this PDE from using the Laplace transform. You may keep your solution in the frequency domain, since the inverse transform may be somewhat complex. Please try to simplify as much as possible using functions like sinh and cosh.

Why can't this PDE be solved by separation of variables? (i.e., try to solve with separation of variables until you hit a contradiction).
