[redacted]

$\boxed{5\text{-}1}$. $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$$\vec{n} = \vec{a} \times \vec{b} = \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = (a_2b_3 - a_3b_2)\vec{i} - (a_1b_3 - a_3b_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$$

$\vec{a} \cdot \vec{n}$ is 0 when vectors are orthogonal

cancel

$$\vec{a} \cdot \vec{n} = a_1(a_2b_3 - a_3b_2) - a_2(a_1b_3 - a_3b_1) + a_3(a_1b_2 - a_2b_1)$$

cancel　　　　cancel

$\vec{a} \cdot \vec{n} = 0$ $\therefore$ orthogonal

check $\vec{b} \cdot \vec{n}$

cancel

$$\vec{b} \cdot \vec{n} = b_1(a_2b_3 - a_3b_2) - b_2(a_1b_3 - a_3b_1) + b_3(a_1b_2 - a_2b_1)$$

cancel

cancel

$\vec{b} \cdot \vec{n} = 0$ $\therefore$ orthogonal

[redacted]

5-2 show $\nabla \cdot (\nabla \times f) = 0$

$$f = a\vec{i} + b\vec{j} + c\vec{k}$$

$$\vec{n} = \nabla \times f = \begin{vmatrix} \vec{i} & \vec{j} & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a & b & c \end{vmatrix}$$

$$= \left(\frac{\partial c}{\partial y} - \frac{\partial b}{\partial z}\right)\vec{i} - \left(\frac{\partial c}{\partial x} - \frac{\partial a}{\partial z}\right)\vec{j} + \left(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y}\right)K$$

cancel cancel

$$\nabla \circ \vec{n} = \frac{\partial}{\partial x}\left(\frac{\partial c}{\partial y} - \frac{\partial b}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial c}{\partial x} - \frac{\partial a}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y}\right)$$

cancel

$$\boxed{\nabla \circ \vec{n} = 0 = \nabla \cdot (\nabla \times f)}$$

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[redacted]

5-3 $f(x,y) = e^{(x^2+y^2)}$ $\quad 1 \leq x^2+y^2 \leq 2$

a) find volume, W

$$x^2+y^2 = r^2$$

$$f(r,\theta) = e^{(r^2)} = z \qquad 1 \leq r^2 \leq 2$$

$$W = \int_1^{\sqrt{2}}\int_0^{2\pi} re^{r^2}\,dr\,d\theta$$

$$= \int_1^{\sqrt{2}} re^{r^2}\theta\Big|_0^{2\pi} dr$$

$$= \int_1^{\sqrt{2}} 2\pi re^{r^2} dr = 2\pi\left(\frac{1}{2}e^{r^2}\Big|_1^{\sqrt{2}}\right)$$

$$\boxed{W = \pi(e^2-e)}$$

b) $\vec{F} = (2x-xy)\vec{i} - y\vec{j} + yz\vec{k}$

$\iiint_V \nabla\cdot\vec{F}\,dV$ = flux of vector field out of W

$$\nabla\cdot\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = 2-y-1+y = 1$$

$$\iiint_V 1\,dV = V = \boxed{\pi(e^2-e)}$$

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[redacted]

5-4

a) $\langle \vec{f}, \vec{g} \rangle = \int_0^1 f(x)g(x)\,dx$

equivalent

$\langle \vec{g}, \vec{f} \rangle = \int_0^1 g(x)f(x)\,dx$ so $\underline{\langle \vec{f}, \vec{g} \rangle = \langle \vec{g}, \vec{f} \rangle}$

$\langle a\vec{f}, \vec{g} \rangle = \int_0^1 af(x)g(x)\,dx$ a=constant

$= a\int_0^1 f(x)g(x)\,dx = a\langle \vec{f}, \vec{g} \rangle$

$\boxed{\therefore \langle a\vec{f}, \vec{g} \rangle = a\langle \vec{f}, \vec{g} \rangle}$

$\langle \vec{f}, \vec{f} \rangle = \int_0^1 f(x)f(x)\,dx = \int_0^1 (f(x))^2\,dx$

since $\vec{f}$ is real valued, always positive unless $\vec{f}$ is 0, the value is 0

$\boxed{\langle \vec{f}, \vec{f} \rangle \geq 0}$

b) $\vec{f} = \cos(\pi mx)$ $\vec{g} = \cos(\pi nx)$

$m > 0$
$n > 0$
$m \neq n$.

$\langle \vec{f}, \vec{g} \rangle = \int_0^1 \cos(\pi mx)\cos(\pi nx)\,dx$

$\cos(\pi mx)\cos(\pi nx) = \frac{1}{2}\left(\cos(\pi mx - \pi nx) + \cos(\pi mx + \pi nx)\right)$

$\langle \vec{f}, \vec{g} \rangle = \frac{1}{2}\left[\int_0^1 \cos(\pi mx - \pi nx)\,dx + \int_0^1 \cos(\pi mx + \pi nx)\,dx\right.$

[redacted]

5-4b

$$\langle \vec{f}, \vec{g} \rangle = \frac{1}{2}\left(\frac{1}{\pi(m-n)} \left(\sin(\pi m x - \pi n x)\right)\Big|_0^1 + \frac{1}{\pi(m+n)} \left(\sin(\pi m x + \pi n x)\right)\Big|_0^1 \right)$$

$\sin(0) = 0$

$\sin(n\pi) = 0 \quad n = \text{integer}$

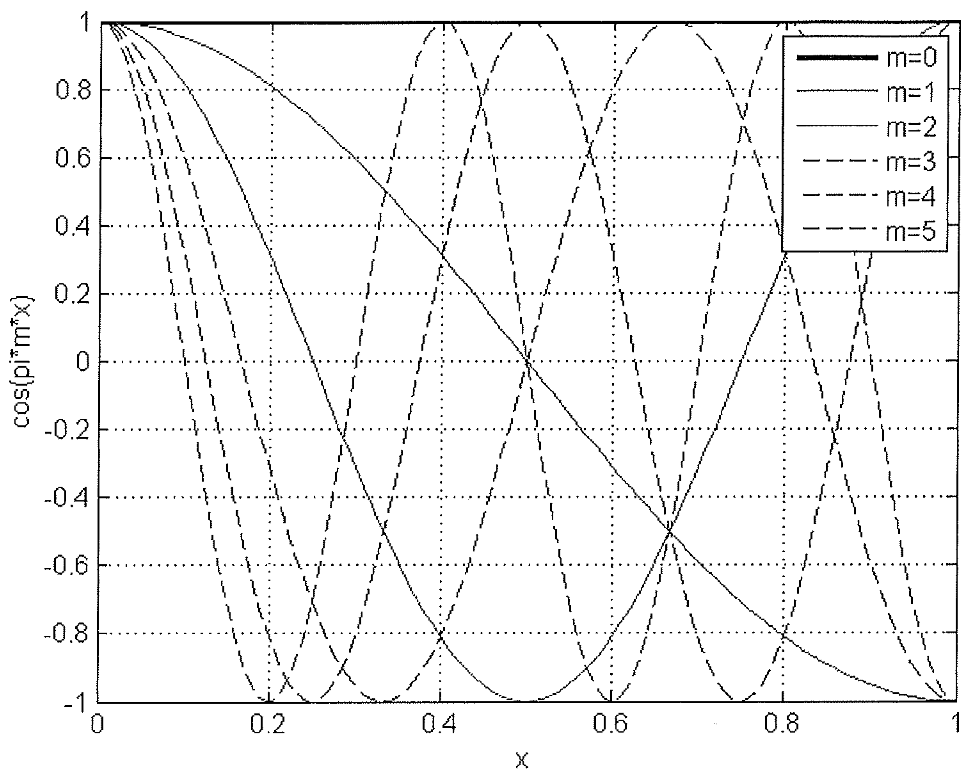
$\therefore$ integrations are 0

$\langle \vec{f}, \vec{g} \rangle = 0$ for $f = \cos(\pi m x)$, $g = \cos(\pi n x)$

$\therefore$ orthogonal

[redacted]

Problem 5c Plot cos(πmx)



Verify orthogonality for cos(πmx) and cos(πnx) for (m,n) pairs:

(1,4)

(2,6)

(3,15)

```
clear all, close all, clc

dx = 0:.01:1;

%For m,n pair (1,4)
m=1;
n=4;
f=cos(pi*m*dx);
g=cos(pi*n*dx);

A=trapz(dx,f.*g)

%For m,n pair (2,6)
m=2;
```

```
n=6;
f=cos(pi*m*dx);
g=cos(pi*n*dx);

B=trapz(dx,f.*g)

%For m,n pair (3,15)
m=3;
n=15;
f=cos(pi*m*dx);
g=cos(pi*n*dx);

C=trapz(dx,f.*g)
```

Command Window Output:

```
A =

  4.5103e-17

B =

 -3.4694e-17

C =

 -1.1796e-16
```