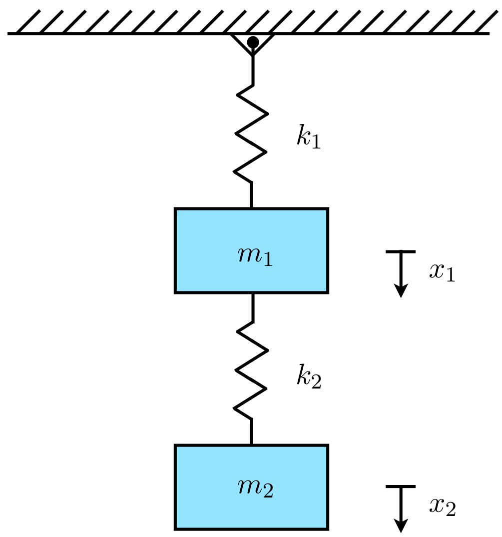
Exercise 2-1 Please compute an analytic expression, by hand, for the real and imaginary parts of the following complex functions. Please also plot these in Matlab for `t=0:.01:10`.

(a) $f(t) = 5e^{(-1+i)t}$,

(b) $f(t) = e^{(1-i)t}$.

Exercise 2-2 Consider the double spring-mass system presented in class (equations below).



$$m_1\ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = 0$$
$$m_2\ddot{x}_2 + k_2 (x_2 - x_1) = 0$$

Write these coupled second order linear differential equations as

(a) A single fourth order ODE in x_1

(b) A single fourth order ODE in x_2

(c) A system of four coupled linear ODEs in terms of the positions and velocities of each mass. Please write this as a matrix ODE.

For the remaining parts, you may assume that $k_1 = k_2 = m_1 = m_2 = 1$. What are the eigenvalues of the matrix system of ODEs in part (c)? You can use Matlab to compute these. Use Matlab to simulate the system from $t = 0$ to $t = 15$ using `ode45` with a sufficiently small time-step, and plot the results; start with an initial condition $x_1(0) = x_2(0) = 1$ and $\dot{x}_1(0) = \dot{x}_2(0) = 0$. Do the simulations agree with your intuition based on the eigenvalues?

Exercise 2-3 Consider the spring-mass-damper system given by:

$$\ddot{x} + 3\dot{x} + 2x = 0$$

Write this as a matrix system of first order ODEs: $\dot{\mathbf{y}} = \mathbf{A}\mathbf{y}$. Compute the eigenvalues and eigenvectors of $\mathbf{A}$ by hand. Write down the solution $\mathbf{y}(t)$ using the matrix exponential in terms of the eigenvalues and eigenvectors. You may write the solution in terms of a general initial condition.

Exercise 2-4 Write the following ODE as a system of first order ODEs:

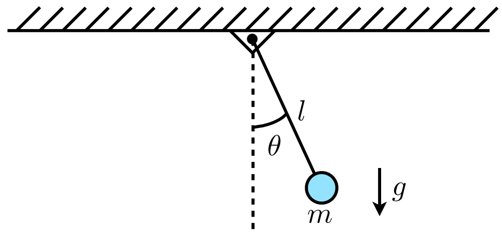
$$x^{(4)} + 5\dddot{x} + 5\ddot{x} - 5\dot{x} - 6x = 0$$

What are the eigenvalues of the system of ODEs? (it is OK to use Matlab)

What are the long-time behaviors of this system for the following two different initial conditions:
$\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$?

Please plot the response for `t=0:0.01:10` using `ode45` for each of these initial conditions. Do the Matlab plots agree with your calculations? Please briefly explain your observations in 1-2 sentences.

Exercise 2-5 Consider the schematic of the single pendulum.



The kinetic energy T and potential energy V may be written as:

$$T = \frac{1}{2} m l^2 \dot{\theta}^2$$
$$V = -gml\cos(\theta)$$

The Lagrangian $\mathcal{L}$ is given by $\mathcal{L} = T - V$, and the Euler-Lagrange equations for the motion of the pendulum are given by the following second order differential equation in θ:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

Write down the second order ODE using the specific T and V defined above. Please write this ODE in the form $\ddot{\theta} = f(\theta, \dot{\theta})$. Notice that this ODE is not linear!!

Now you may assume that $l = m = g = 1$ for the remainder of the problem.

You may still suspend variables to get a system of two first order (nonlinear) ODEs by writing the ODE as:

$$\dot{\theta} = \omega$$
$$\dot{\omega} = f(\theta, \omega)$$

What are the fixed points of this system where all derivatives are zero?

Write down the linearized equations in a neighborhood of each fixed point and determine the linear stability. You may formally linearize the nonlinear ODE or you may use a small angle approximation for $\sin(\theta)$; the two approaches are equivalent.