ME564 - Autumn 2018 **Due: 5:00:00pm Friday October 19^{th}, 2018**
Homework 2

Exercise 2–1: Consider the following ODE:

$$\ddot{x} + 10\dot{x} + 21x = 0.$$

(a) Solve this ODE by substituting in $x(t) = e^{\lambda t}$ and finding roots of the characteristic polynomial.

(b) Now, write this second order ODE as a system of first order equations, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, by introducing a new variable $v = \dot{x}$. Solve the linear system by decomposing the matrix $\mathbf{A}$ into the eigenvalues and eigenvectors. Hint: computing the inverse of the matrix of eigenvectors is easier if they are not normalized (i.e., both of my eigenvectors have integer entries).

(c) For both of the solutions in parts (a) and (b), write down the specific solution for an initial condition $x(0) = 2$ and $\dot{x}(0) = -10$.

(d) What is the long-time behavior of this system? How would this change if the equation was $\ddot{x} - 10\dot{x} + 21x = 0$?

Exercise 2–2:

(a) Write the following ODE as a system of first order ODEs:

$$\dddot{x} + 2\ddot{x} - \dot{x} - 2x = 0$$

What are the eigenvalues of the system of ODEs? (it is OK to use Matlab).

What are the long-time behaviors of this system for the following two different initial conditions: $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$?

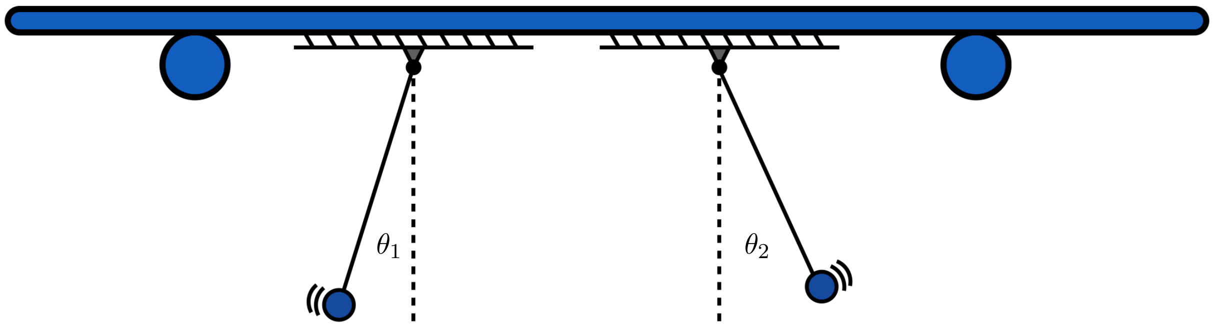
Please plot the response for `t=0:0.01:10` using `ode45` for each of these initial conditions. Do the Matlab plots agree with your calculations? Please briefly explain your observations in 1-2 sentences.

(b) Repeat the steps above for the ODE

$$x^{(4)} + 5\dddot{x} + 5\ddot{x} - 5\dot{x} - 6x = 0$$

with initial conditions: $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \dddot{x}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

Exercise 2–3: Consider the system of weakly coupled pendulua (equations below). Both pendula are mounted to a board that is placed on rollers, so that it can move from side to side, slightly.



$$\ddot{\theta}_1 = -\omega_1^2\theta_1 + \epsilon(\theta_2 - \theta_1)$$
$$\ddot{\theta}_2 = -\omega_2^2\theta_2 + \epsilon(\theta_1 - \theta_2).$$

Write these coupled second order linear differential equations as

(a) A single fourth order ODE in θ_1

(b) A single fourth order ODE in θ_2

(c) A system of four coupled linear ODEs in terms of the angular positions and velocities of each pendulum. Please write this as a matrix ODE.

(d) Now, assume that $\omega_1 = 1$ and $\omega_2 = 1.5$. Increase ϵ from 0 to 0.5 (in increments of 0.005), and compute the eigenvalues of the system of equations. Plot the two frequencies as a function of ϵ. Now plot the difference of the two frequencies agains ϵ. Explain what you see.

(e) At what value of ϵ will the frequencies of the coupled system be equal for $\omega_1 = 1$ and $\omega_2 = 1.5$?

(f) At what value of ϵ will the frequencies of the coupled system be equal for generic ω_1 and ω_2?

Exercise 2–4: In this exercise, we will represent the solution of a nonlinear differential equation in terms of an infinite power series expansion. Consider the differential equation:

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - a^2\right)y = 0.$$

Derive a formula for the coefficients of the power series expansion for $y(x)$ for the case where $a = 1$.
