ME564 - Autumn 2018 **Due: 5:00:00pm Friday October 5^{th}, 2018**
Homework 1

Exercise 1–1: Compute the derivative of the following functions

(a) $f(x) = \cos(x^3)$

(b) $f(x) = 2^x$

(c) $f(x) = e^{x^2} \sin(x)$

(d) $f(x) = \tan(x^2)$

(e) Compute the derivative of $f(x, y) = \cos(x^2) \sin(y^2)$ with respect to t, assuming that $x(t)$ and $y(t)$ vary with time. You can write the solution in terms of dx/dt and dy/dt.

Exercise 1–2: Consider a vat that is mixing salt water. The vat initially contains a 100 liter mixture with 5 kg of salt. At time $t = 0$ a mixture containing 0.1 kg/liter is added to the vat at a rate of 0.1 liters per second. The solution is well-mixed in the tank, and liquid is drained at a rate of 0.1 liters per second.

(a) Let $x(t)$ denote the mass of salt in the vat as a function of time. Write down a differential equation for $x(t)$ and its solution.

(b) Plot the concentration of salt in the vat as a function of time.

(c) How long until there are 7 kg of salt in the vat?

(d) How much salt is in the vat after 30 seconds?

(e) How much salt is in the vat after a very long time?

Exercise 1–3: Compute the Taylor series expansion by hand for $f(x)$. For each function, plot $f(x)$ and the three-term expansion (i.e., the first three nonzero terms) from $x = -5$ to $x = 5$.

(a) $f(x) = x \cos(x)$.

(b) $f(x) = \cos(x)/x^2$.

(c) $f(x) = e^{x^2}$.

(d) $f(x) = \frac{1}{1-x}$; when is this series valid?

(e) $f(x) = \sin(1/x)$; when is this series valid?

Exercise 1–4: Write down the solution to the following differential equation (by hand):

$$\ddot{x} - \lambda x = 0,$$

for an arbitrary initial position $x(0)$ and zero velocity $\dot{x}(0)$. What does the solution look like for $\lambda > 0$? How about for $\lambda < 0$? Please sketch solutions.

Exercise 1–5: Please solve the following differential equation (by hand) and describe the long-time behavior of the system:

(a) $\ddot{x} + 4\dot{x} + 3x = 0$, with initial conditions $x(0) = 0$ and $\dot{x}(0) = 4$.

(b) $\ddot{x} - 4x = 0$, with initial conditions $x(0) = 4$ and $\dot{x}(0) = -4$.

Exercise 1–6: Write down the second-order differential equation $\ddot{x} + K_1\dot{x} + K_2 x = 0$ that has eigenvalues $\lambda = C$ and $\lambda = D$. If the solution is $x(t) = Ae^{Ct} + Be^{Dt}$, what are the initial conditions?

Exercise 1–7: Please compute an analytic expression, by hand, for the real and imaginary parts of the following complex functions. Please also plot these in Matlab for `t=0:.01:10`.

(a) $f(t) = e^{-t}$,

(b) $f(t) = e^{t}$,

(c) $f(t) = e^{2\pi i t}$,

(d) $f(t) = e^{i(2\pi t + \pi/2)}$,

(e) $f(t) = e^{t(-1+i)}$,

(f) $f(t) = e^{t(1+i)}$.
