ME564 - Autumn 2017 **Due: 11:59pm Friday November 17^{th}, 2017**
Homework 4

Exercise 4-1 Consider the differential equation of the periodically forced Duffing oscillator (use `ode45`):

$$\ddot{x} = -\delta\dot{x} - \beta x - \alpha x^3 + \gamma\cos(\omega t)$$

We may write this as a system of first order differential equations as

$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= -\delta v - \beta x - \alpha x^3 + \gamma\cos(\omega t)\end{aligned}$$

These equations may be used to model a periodically forced beam that is deflected towards two magnets. This was one of the first examples that was used to demonstrate chaos (see `http://www.scholarpedia.org/article/Duffing_oscillator` for historical details and references).

For all of the parts, use $\alpha = 1, \beta = -1$, and $d = 0.2$. You will want to create a MATLAB .m file called `duffing.m` with the following inputs and outputs:

```
function dy = duffing(t,y,a,b,d,g,w)
```

Here `a` is α, `b` is β, `d` is δ, `g` is γ and `w` is ω.

(a) Integrate this ODE with initial condition $x = 2$ and $v = 0$ from $t = 0$ to $t = 100$ with $\Delta t = 0.01$. First, integrate the unforced case with $\gamma = 0$. Plot the solution of x vs t.

(b) Now, lets use a forcing frequency of $\omega = 1$ and increase γ from 0.1 to 0.5 in increments of 0.1. For each of the five values of γ, plot x and v against t. As you change γ, note that chaos ensues!

Exercise 4-2 Derive a $\mathcal{O}(\Delta t^2)$ accurate forward difference derivative for the function $f(t)$ at t using the following three *measurements* or data points:

$$f(t), f(t+\Delta t), \text{ and } f(t+2\Delta t)$$

Please show that this scheme is really $\mathcal{O}(\Delta t^2)$ accurate using the Taylor series expansions.

Exercise 4-3 Consider a column vector of data $\mathbf{f}(t)$ with 6 elements. Write down the 6×6 matrix $\mathbf{D}$ that when multiplied by $\mathbf{f}$ will yield a vector of derivatives of $\mathbf{f}$ with respect to time:

$$\frac{d\mathbf{f}}{dt} = \mathbf{D}\mathbf{f}.$$

You may assume that all of the data in $\mathbf{f}$ was collected $\Delta t = 0.1$ time units apart. Please use forward difference for the first point, backward difference for the last point, and central difference for the four interior points.

Exercise 4-4

Lets say that we are integrating the following stable ODE:

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) If we are using a forward Euler scheme, for what $\Delta t > 0$ will the resulting simulation become unstable?

(b) If we use the backward Euler scheme, for what $\Delta t > 0$ will the simulation become unstable?