ME564 - Autumn 2017 **Due: 11:59 pm Friday November 3^{rd}, 2017**
Homework 3

Exercise 2–1: Please solve the following differential equation (with initial conditions) for the three cases below (by hand!). You may use whatever method you find simplest. You may check your work in MATLAB.

$$\begin{aligned}
&\ddot{x} + 5\dot{x} + 6x = f(t),\\
&x(0) = \frac{1}{2},\\
&\dot{x}(0) = -1.
\end{aligned}$$

(a) For $f(t) = 0$. Note that this is just the unforced ODE $\ddot{x} + 5\dot{x} + 6x = 0$.

(b) For $f(t) = e^{-t}$.

(c) For $f(t) = 50\cos(t)$. Hint: try a particular solution $x_P = A\cos(t) + B\sin(t)$ and solve for A and B.

In all cases, be sure to make sure that your initial conditions are still satisfied!

Exercise 2–2: Solve the following linear system of ODEs:

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x}, \quad \text{for} \quad \mathbf{A} = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

Please compute this by hand for a generic initial condition $\mathbf{x}(0) = \mathbf{x}_0$. Hint: you will want to compute the matrix exponential $e^{\mathbf{A}t}$. You may use MATLAB to check your work.

Exercise 2–3: Consider the ODE for the pendulum:

$$\ddot{\theta} = -\sin(\theta)$$

What is the stability of the inverted position, when $\theta = \pi$?

Now, we are going to add stabilizing feedback control, assuming that we can measure θ and $\dot{\theta}$:

$$\ddot{\theta} = -\sin(\theta) + \tau$$

where $\tau = -2\dot{\theta} - 2(\theta - \pi)$ is an applied torque at the base of the pendulum.

What is the new stability of the inverted position, when $\theta = \pi$?

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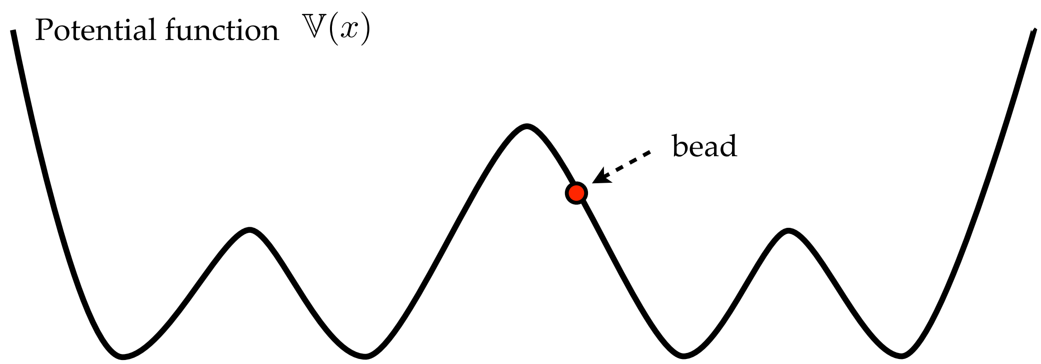
Exercise 2–4: Consider the following physical system of a bead constrained to move in the potential field $\mathbb{V}(x)$. Note: there are no equations given, and so you must use physical intuition.

- Please sketch the phase portrait (position x vs. velocity v) for the Please try to make your sketch as accurate as possible using as much information about the potential as you can. You may draw the phase portrait without any damping (i.e., no friction).
- Comment briefly on how the plot will change if we turn on friction. (you do not need to sketch anything)
- How many fixed points does the system have and what is their stability?
- What can we say about the eigenvalues of the linearization around each point?
- Finally, please pick a trajectory on the phase portrait and explain in one sentence what this trajectory means physically.

Quadruple potential well.



Phase portrait