Exercise 1–1: Compute the derivative of the following functions

(a) $f(x) = \cos(x^3)$

(b) $f(x) = x^x$

(c) $f(x) = e^{\sin(2x)} \cos(x)$

(d) Now, compute the derivative of $f(x, y) = \cos(x^2 + y^2)$ with respect to t, assuming that $x(t)$ and $y(t)$ vary with time. You can write the solution in terms of dx/dt and dy/dt.

Exercise 1–2: A given mass x of a radioactive element obeys the following differential equation in time:

$$\dot{x} = \lambda x,$$

where λ is a constant describing the rate of decay.

(a) Write down the solution $x(t)$ to the differential equation.

(b) Plot the solution for an initial condition $x(0) = 2$ from time $t = 0$ to $t = 5$ for $\lambda = -5, -1, 0, 0.01, 0.1$. Please plot these all on the same figure using the `hold on` command in Matlab. Label your axes (`>> doc xlabel`, `>> doc ylabel`) and include a legend (e.g., `legend('lambda 1','lambda 2','lambda 3', ...)`).

(c) The half-life T is defined as the time it takes for the material to be reduced to half of its mass through radioactive decay. The half-life of uranium-238 is 4.468 billion years. What is the corresponding value of λ?

(d) If you start with $100kg$ of uranium-238, how long until you only have $5kg$ left?

Exercise 1–3: Compute the Taylor series expansion by hand for $f(x)$. For each function, plot $f(x)$ and the three-term expansion (i.e., the first three nonzero terms) from $x = -5$ to $x = 5$.

(a) $f(x) = \sin(x)/x$.

(b) $f(x) = 3^x$.

Exercise 1–4: Write down the solution to the following differential equation (by hand):

$$\ddot{x} - \lambda x = 0,$$

for an arbitrary initial position $x(0)$ and zero velocity $\dot{x}(0)$. What does the solution look like for $\lambda > 0$? How about for $\lambda < 0$? Please sketch solutions.

Exercise 1–5: Please solve the following differential equation (by hand) and describe the long-time behavior of the system:

(a) $\ddot{x} + \dot{x} - 6x = 0$, with initial conditions $x(0) = 5$ and $\dot{x}(0) = -5$.

(b) $\ddot{x} + 8\dot{x} + 15x = 0$, with initial conditions $x(0) = 3$ and $\dot{x}(0) = -11$.

Exercise 1–6: Please compute an analytic expression, by hand, for the real and imaginary parts of the following complex functions. Please also plot these in Matlab for `t=0:.01:10`.

(a) $f(t) = e^{it}$,

(b) $f(t) = e^{(-1-i)t}$,

(c) $f(t) = e^{1-it}$.

(d) $f(t) = e^{(-.2+3\pi i)t}$.
