Mechanical Engineering 564
Midterm Exam
Fall Quarter 2018

Question	Score
1	/ 25
2	/ 25
3	/ 25
4	/ 25
Total	/100

Name:

UW NetID:

UWStudent ID:

Instructions: This is a 4 hour, take-home exam, and everybody is expected to abide by the honor code and follow the rules below. The 4 hours you use for the exam do not need to be consecutive. For example, you can work on the exam for 2 hours on Wednesday and another 2 hours on Thursday. Please be reasonable, and try not to split the exam up into more than four blocks of time.

You are allowed to use the course notes (online pdfs and your own handwritten notes), online ME564 lecture videos, and *your own* homework solutions on the exam. All other resources are prohibited, including: the internet, other books, discussing the exam with other people.

Some problems will allow or encourage the use of MATLAB. It will be specifically mentioned in the problem if MATLAB is allowed. Otherwise, please do not use computers on the exam.

Bona Fortuna!!

ME564 - Autumn 2018 **Due: 5:00:00pm Friday October 26^{th}, 2018**
Midterm Exam

Exercise 1: Find the solution of the following linear differential equation:

$$\dddot{x} - 7\dot{x} - 6x = 0 \tag{1}$$

for the following initial conditions: $x(0) = 6, \dot{x}(0) = 4, \ddot{x}(0) = 36$.

Also, write the equation in matrix form as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ for $\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$.

Show that the characteristic equation for Eq. (1) is the same as the eigenvalue equation of $\mathbf{A}$. You may solve for the specific eigenvalues of $\mathbf{A}$ in Matlab, but work out the rest of the problem by hand.

Is the long time behavior stable or unstable? Are there any initial conditions that would result in asymptotically stable behavior? If so, please give one of these initial conditions.

Exercise 2: Find the general solution of the following linear differential equation:

$$\ddot{x} + 6\dot{x} + 5x = 0. \tag{2}$$

Are the dynamics stable or unstable?

Find the specific solution for the following initial conditions:

- $x(0) = 3$ and $\dot{x}(0) = -11$
- $x(0) + \dot{x}(0) = -4$ and $x(0) - \dot{x}(0) = 8$
- $x(1) = 2$ and $\dot{x}(1) = -6$

Exercise 3: In this exercise, we will represent the solution of a nonlinear differential equation in terms of an infinite power series expansion. Consider the differential equation:

$$\frac{d^2y}{dx^2} - xy = 0.$$

Derive a formula for the coefficients of the power series expansion for $y(x)$.

Exercise 4: Consider the following differential equation:

$$\ddot{x} - b\dot{x} + ax = 0.$$

What are all of the possible behaviors of the system based different values of a and b? Draw a two dimensional diagram in the $a - b$ plane and highlight regions where the solutions behave similarly. Label these regions (e.g., "saddle", "center", etc.), indicate what the eigenvalues are, and draw a sketch for the behavior in each region.