Mechanical Engineering 564
Final Exam
Fall Quarter 2018

Question	Score
1	/ 25
2	/ 15
3	/ 20
4	/ 20
5	/ 20
Total	/100

Name:

UW NetID:

UWStudent ID:

Instructions: This is a 4 hour, take-home exam, and everybody is expected to abide by the honor code and follow the rules below. The 4 hours you use for the exam do not need to be consecutive. For example, you can work on the exam for 2 hours on Wednesday and another 2 hours on Thursday. Please be reasonable, and try not to split the exam up into more than four blocks of time.

You are allowed to use the course notes (online pdfs and your own handwritten notes), online ME564 lecture videos, and *your own* homework solutions on the exam. All other resources are prohibited, including: the internet, other books, discussing the exam with other people.

Some problems will allow or encourage the use of MATLAB. It will be specifically mentioned in the problem if MATLAB is allowed. Otherwise, please do not use computers on the exam.

Bona Fortuna!!

Exercise 1: Please solve the following short problems:

(a) Compute the derivative of $f(x) = 2^{\cos(x)}$.

(b) Write down the real and imaginary parts of $f(t) = e^{(-1+i\pi/2)t}$.

(c) Write down the Taylor series for $f(x) = \sin(x^2)$.

(d) Write the solution $x(t)$ for $\ddot{x} + 6\dot{x} + 8x = 0$, with initial conditions $x(0) = 3$ and $\dot{x}(0) = -8$.

(e) Consider the following finite difference scheme: $\frac{1}{8\Delta x}\left(f(x + 4\Delta x) - f(x - 4\Delta x)\right)$. What derivative is this approximating? What is the order of accuracy of this scheme?

Exercise 2: Please solve the following differential equation (with initial conditions) for the three cases below (by hand!). You may use whatever method you find simplest. You may check your work in MATLAB.

$$\begin{aligned} &\ddot{x} + \dot{x} - 2x = f(t), \\ &x(0) = 24, \\ &\dot{x}(0) = -12. \end{aligned}$$

(a) For $f(t) = 0$. Note that this is just the unforced ODE $\ddot{x} + \dot{x} - 2x = 0$.

(b) For $f(t) = e^{-3t}$.

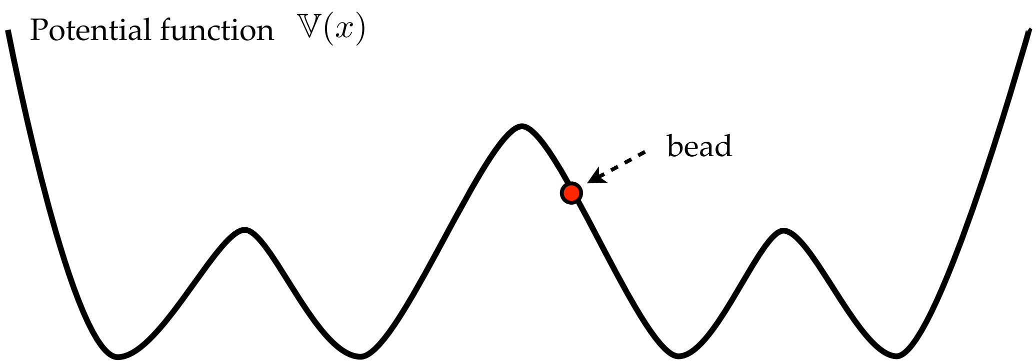
(c) For $f(t) = 30\sin(t)$. Hint: try a particular solution $x_P = A\cos(t) + B\sin(t)$ and solve for A and B.

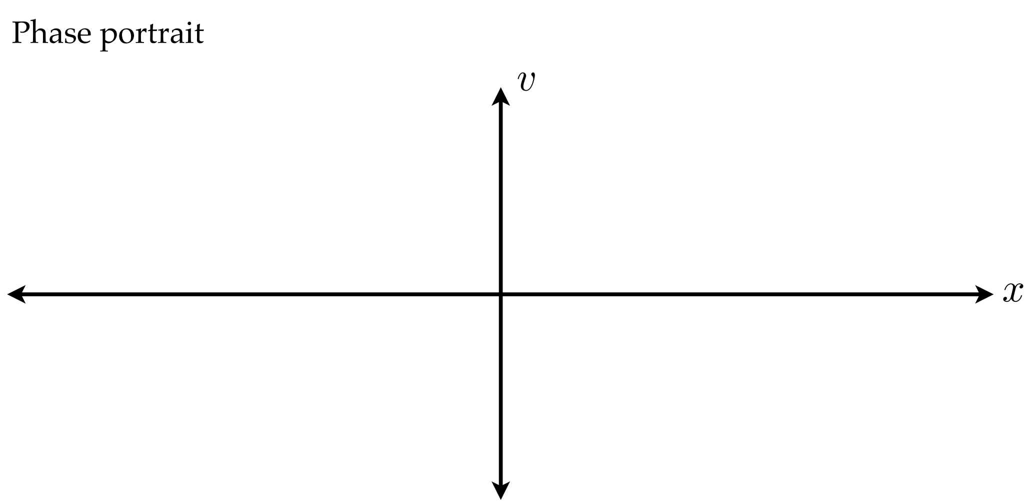
In all cases, be sure to make sure that your initial conditions are still satisfied!

Exercise 3: Consider the following physical system of a bead constrained to move in the potential field $\mathbb{V}(x)$. Note: there are no equations given, and so you must use physical intuition.

- Please sketch the phase portrait (position x vs. velocity v) for the Please try to make your sketch as accurate as possible using as much information about the potential as you can. You may draw the phase portrait without any damping (i.e., no friction).
- Comment briefly on how the plot will change if we turn on friction. (you do not need to sketch anything)
- How many fixed points does the system have and what is their stability?
- What can we say about the eigenvalues of the linearization around each point?
- Finally, please pick a trajectory on the phase portrait and explain in one sentence what this trajectory means physically.

Quadruple potential well.





Exercise 4: Consider the following nonlinear, forced ODE:

$$\ddot{x} + \delta\dot{x} + \beta x + \alpha x^3 = \gamma\cos(\omega t)$$

For $\beta = -1$, $\alpha = 1$ and $\gamma = 0$, we may write this as a system of first order differential equations as

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -\delta v + x - x^3 \end{aligned}$$

You may assume that $\delta \geq 0$.

(a) Write down all of the fixed points of the system of equations.

(b) For each fixed point, write down the linearized equations near the fixed point.

(c) For each linearized system, set $\delta = 0$ and determine what type of fixed point it is (source, sink, center, spiral, saddle, etc.) and what the stability is.

(d) Describe in words how these fixed points will change if δ is a small positive number.

(e) Please sketch the phase portrait (i.e. trajectories in the x-v plane) for the system with a small positive δ. Pick one of the stable fixed points and sketch the set of initial conditions that will eventually end up near this fixed point.

Exercise 5: Please solve the following:

(a) Show that $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and $\mathbf{b}$.

(b) You are goat climbing a mountain, where the altitude is given by the function $f(x, y) = 10 - x^3 + y^2$. To impress the humans below, you want to choose the steepest path up the mountain. You are at the point $(x, y) = (1, 2)$. What direction should you go in to climb the steepest path?

(c) You are a cold mosquito flying around a room. The temperature is given by $T(x, y, z) = \sin(x)\cos(y)\sin(z)$, and you are at the point $(1, 2, 2)$. What direction should you fly to warm up?
