Mechanical Engineering 564
Midterm Exam
Fall Quarter 2017

Question	Score
1	/ 25
2	/ 25
3	/ 20
4	/ 15
5	/ 15
Total	/100

Name:

UW NetID:

UWStudent ID:

Instructions: This is a 4 hour, take-home exam, and everybody is expected to abide by the honor code and follow the rules below. The 4 hours you use for the exam do not need to be consecutive. For example, you can work on the exam for 2 hours on Wednesday and another 2 hours on Thursday. Please be reasonable, and try not to split the exam up into more than four blocks of time.

You are allowed to use the course notes (online pdfs and your own handwritten notes), online ME564 lecture videos, and *your own* homework solutions on the exam. All other resources are prohibited, including: the internet, other books, discussing the exam with other people.

Some problems will allow or encourage the use of MATLAB. It will be specifically mentioned in the problem if MATLAB is allowed. Otherwise, please do not use computers on the exam.

Bona Fortuna!!

ME564 - Autumn 2017 **Due: 5:00:00pm Friday December 8^{th}, 2017**
Final Exam

Exercise 1: Please solve the following differential equation (with initial conditions) for the three cases below (by hand!). You may use whatever method you find simplest. You may check your work in MATLAB.

$$\begin{aligned}&\ddot{x} + 3\dot{x} + 2x = f(t),\\ &x(0) = 2,\\ &\dot{x}(0) = -3.\end{aligned}$$

(a) For $f(t) = 0$. Note that this is just the unforced ODE $\ddot{x} + 3\dot{x} + 2x = 0$.

(b) For $f(t) = 2e^{-3t}$.

(c) For $f(t) = 20\sin(t)$. Hint: try a particular solution $x_P = A\cos(t) + B\sin(t)$ and solve for A and B.

In all cases, be sure to make sure that your initial conditions are still satisfied!

Exercise 2: Consider the differential equation for the Van der Pol oscillator (use `ode45`)

$$\ddot{y} + \epsilon(y^2 - 1)\dot{y} + y = 0$$

which has a nonlinear damping term $\epsilon(y^2 - 1)\dot{y}$.

(a) Write this ODE as a system of first order differential equations.

(b) Analyze the stability of the fixed point at $y = \dot{y} = 0$ for $\epsilon > 0$.

(c) For $\epsilon = 0.1$, solve the equation for `t=0:0.1:30` for initial conditions $y(0) = 0.1$ and $\dot{y}(0) = -1$. Repeat for $\epsilon = 1$ and $\epsilon = 20$. Plot the phase portrait (y vs. $\dot{y}$) for each of the three cases to see the various behaviors.

You will want to create a MATLAB .m file called `vanderpol.m` with the following inputs and outputs:

```
function dy = vanderpol(t,y,eps)
```

Exercise 3: Here we will compute using the directional derivative $D_{\mathbf{v}} f = \nabla f \cdot \mathbf{v}/\|\mathbf{v}\|$.

(a) Compute the directional derivative with $f(x, y) = x^3 \cos(xy)$ and $\mathbf{v} = \begin{bmatrix} 2 & 1 \end{bmatrix}^T$.

(b) You are goat climbing a mountain, where the altitude is given by the function $f(x, y) = 5 - x^2 - y^6$. To impress the humans below, you want to choose the steepest path up the mountain. You are at the point $(1, 1, 3)$. What direction should you go in to climb the steepest path?

(c) You are a cold mosquito flying around a room. The temperature is given by $T(x, y, z) = \cos(x^2)\sin(y)\cos(z)$, and you are at the point $(1, 2, 1)$. What direction should you fly to warm up?

Exercise 4: Show the following:

(a) $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and $\mathbf{b}$.

(b) $\nabla \cdot (\nabla \times \mathbf{f}) = 0$ for any vector field $\mathbf{f}$.

(c) $\nabla^2 \phi = \nabla \cdot (\nabla \phi)$ for any potential function ϕ.

(d) $\nabla \cdot (\mathbf{f} \times \mathbf{g}) = \mathbf{g} \cdot (\nabla \times \mathbf{f}) - \mathbf{f} \cdot (\nabla \times \mathbf{g})$.

Exercise 5: Given vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$, the triple product is defined as $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. Show that the triple product satisfies the following:

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

One immediate consequence is that $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$ for all $\mathbf{a}$ and $\mathbf{b}$.
