

This section is to be taken home, completed, and turned in by 8:00pm Tuesday Feb 7th. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

1. The subgroup lattice of the group $\mathbb{Z}_6 \times \mathbb{Z}_3$ is shown below. The same lattice is given on the right but with the generators (as subgroups) for each subgroup.
 - (a) [4] Identify what kind of algebraic object $\mathbb{Z}_6 \times \mathbb{Z}_3$ is. (Ring? Commutative Ring? with Identity? Division Ring? Integral Domain? UFD? PID? ED? Field?) Briefly justify your answer.
 - (b) [2] In the subgroup lattice below, circle any subgroups that are also ideals.
 - (c) [2] In the subgroup lattice below, underline any subgroups that are subrings but not ideals.
 - (d) [2] In the subgroup lattice below, cross out any subgroups that are not subrings.
 - (e) [5] Identify a quotient ring of $\mathbb{Z}_6 \times \mathbb{Z}_3$ that is a field. Write down the elements of the quotient ring and give the addition and multiplication tables.

