Vanderpool

TMath 403

Practice

This exam is intended for practice. The actual exam may look nothing like it.

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. A subring of a field, is a field.

2. Let R be a ring and $r \in R$. The subgroup generated by r is the same as the ideal $\langle r \rangle$.

3. $\mathrm{GL}_2(\mathbb{R}) = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} | a, b, c, d \in \mathbb{R}, ad - bc \neq 0\}$ is a division ring.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

4. For each square below provide an example that satisfies the condition given in the row *and* the condition in the column, if possible. If not possible, briefly explain why.

	not a Commutative Ring	not a Field	not a PID
Commutative Ring			
Integral Domain			

5. Find a unit in $\mathbb{Z}_6[x]$. Justify your answer.

6. Given that 3 is a root, factor $5x^4 + 2x^2 - 3$ in $\mathbb{Z}_7[x]$ into irreducible elements.

7. Let $D = \mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\}$. Let F_D be the field of fractions of D.

(a) Let I be the ideal generated by $\sqrt{2}$. Identify the ring D/I up to isomorphism and prove the isomorphism.

(b) Identify two representatives of the same element in F_D.

(c) Show addition is well defined in F_D

(d) Build a ring homomorphism between D and F_D.

8. [8] Choose *ONE* of the following theorems to prove. Clearly identify which of the two you are proving and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   Theorem 1. *Let $f : R \to S$ be a ring homomorphism between commutative rings. If f is surjective, and I is an ideal of R, show that $f(I)$ is an ideal of S.*

   Theorem 2. *Let R be a ring with a multiplicative identity. If x is a zero divisor then x is not a unit.*