Vanderpool

TMath 403

Winter 2023

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. [3] A subring of a field, is a field.

2. [3] The map $\phi : \mathbb{Z}_4 \to \mathbb{Z}_{12}$ defined by $\phi(x) = 3x$ is a ring homomorphism.

3. [3] The set $\{(x, y, z) \in \mathbb{R}^3 | x - 2y + 2z = 0\}$ is a (vector) subspace.

4. [5] Place the following algebraic objects into a square below so that the object satisfies the condition given in the row *and* the condition in the column. Note, there maybe more than one square that the object fits in, but you need only place each object once.

$\mathbb{Z}$ $\qquad$ $\mathbb{Z}_{12}$ $\qquad$ $\text{Mat}_2(\mathbb{R}) = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} | a, b, c, d \in \mathbb{R}\}$ $\qquad$ $\mathbb{Z}_6 \times \mathbb{Z}_3$ $\qquad$ $\mathbb{Z}[i]$

	not Commutative	not a Field	not a PID
Ring			
Integral Domain			

5. Find an example for each of the following in $\mathbb{Z}[x]$, if possible. No proofs are necessary. Be careful of notation!!

   (a) [1] a principal ideal

   (b) [2] an ideal that is not principal

   (c) [2] a maximal ideal

   (d) [2] a factor ring of R that is also a field

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

6. [3] Given that 4 is a root, factor $x^4 + 4x^3 + x^2 + 2$ in $\mathbb{Z}_5[x]$ into irreducible elements.

7. [8] Choose *ONE* of the following theorems to prove. Clearly identify which of the two you are proving and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

Theorem 1. *Let ϕ be a ring homomorphism from a ring R to a ring S. Let B be an ideal of S. The set $\phi^{-1}(B)$ defined as $\{r \in R | \phi(r) \in B\}$ is an ideal of R.*

Theorem 2. *Let F be a field. Show that the field of quotients of F is ring-isomorphic to F.*