TMath 403

This section is to be taken home, completed, and turned in by 8:00pm Wednesday March 15th. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

1. [3] Construct the field $F = \mathbb{Q}(\sqrt{3}, \sqrt{5})$ and highlight any subfield(s) you create along the way. Note that this construction can be done in different ways to highlight different subfield(s).

2. [2] Identify all subfields of F that are extensions of $\mathbb{Q}$ and arrange these in a lattice.

3. Let $F = \mathbb{Q}(\sqrt{3}, \sqrt{5})$. We write an element of F with the basis $\{1, \sqrt{3}, \sqrt{5}, \sqrt{15}\}$. Define $\tau : F \to F$ by $\tau(a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}) = a-b\sqrt{3}+c\sqrt{5}-d\sqrt{15}$ for $a, b, c, d \in \mathbb{Q}$. Define $\sigma : F \to F$ by $\sigma(a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}) = a+b\sqrt{3}-c\sqrt{5}-d\sqrt{15}$ for $a, b, c, d \in \mathbb{Q}$.

   (a) [4] Verify τ is a field isomorphism.

   (b) [3] Verify the set of isomorphisms from F to F forms a group generated by σ and τ. Provide a Cayley table or Cayley Diagram.

   (c) [2] Create a subgroup lattice for the group above.

   (d) [1] Compare the lattice of (c) to the lattice (2).