Modulo

Our goal is to capture the idea of modulo 4 that came up in the first day of class.

1. "Modulo 4" partitions the integers into equivalence classes.
   For example, $[2] = \{... - 6, -2, 2, 6, 10, ...\}$. List the remaining equivalence classes and identify some integers that are in each equivalence class. (Do you remember/know what an equivalence class is from Chapter 1?!?)

2. Note that we can make use of nice properties (enumerated in Section 3.1) and add equivalence classes together in a consistent way. For example, $[2] + [3] = [5]$ and since $[5] = [1]$, we could write $[2] + [3] = [1]$ (which you might have seen in TMath 300 as written 2+3=1 mod 4). Find the following:

$[0] + [0] =$	$[1] + [0] =$	$[2] + [0] =$	$[3] + [0] =$
$[0] + [1] =$	$[1] + [1] =$	$[2] + [1] =$	$[3] + [1] =$
$[0] + [2] =$	$[1] + [2] =$	$[2] + [2] =$	$[3] + [2] =$
$[0] + [3] =$	$[1] + [3] =$	$[2] + [3] =$	$[3] + [3] =$
$[0] + [4] =$	$[1] + [4] =$	$[2] + [4] =$	$[3] + [4] =$
$[0] + [5] =$	$[1] + [5] =$	$[2] + [5] =$	$[3] + [5] =$

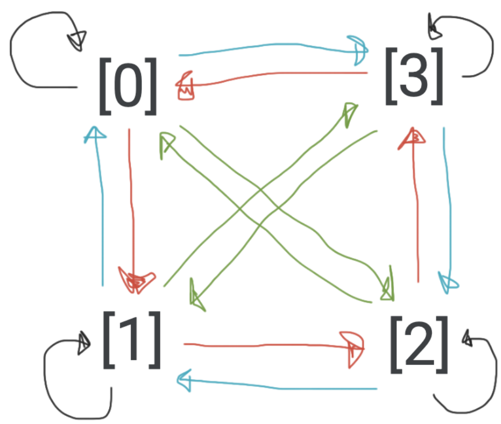
3. Do we need to continue the chart? That is, do we need to have a $[0] + [b]$ row (where b is an integer)? Did we need all the rows that were given? Why or why not?
   (You'll see a more efficient way of writing this chart in Section 3.2.)

4. Let's try to consider capturing this additive structure with a Cayley Diagram.

   First, notice that there are only four equivalence classes (much like there were only four configurations of the pinwheel!). We place all the equivalence classes on the page to act as our vertices.

   Second we add edges between the configurations that correspond to the actions found above. For example, the "plus 0" action can be captured with black arrows shown. Specifically, the class [0] when acted on by "plus 0" returns [0] which is recorded as the black arrow in the upper left of the Cayley Diagram below.

   Identify what the red, green, and blue addition corresponds to.



5. Notice that there is not really anything special about the four equivalence classes and where we put them. For example, we could have put [1] in the upper left spot but the resulting Cayley Diagram would have looked the same. This symmetry lets us drop the labels on the vertices and simplify to that below.

   We can further simplify the below picture by noting that the green arrows ("plus 2 mod 4") are equivalent to two of the red arrows ("plus 1 mod 4"). Can we further reduce the number of arrows in the below diagram by identifying other arrows that are "made up" of existing (generating!) arrows?