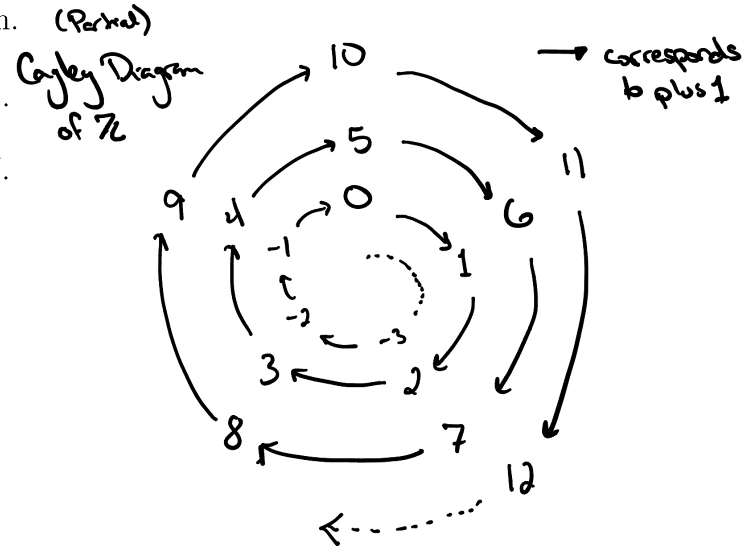
Cosets

Consider $\mathbb{Z}$ with the binary operator of addition. Note that we can arrange the (partial) Cayley Diagram for $\mathbb{Z}$ in a spiral as shown on the right.



1. Let $H = 5\mathbb{Z}$. Identify some elements of H.

2. Show H is a subgroup of $\mathbb{Z}$.

3. Identify some elements in the cosets $1 + H$ and $6 + \mathbb{Z}$ in $\mathbb{Z}$.

4. Note that the "1" and the "6" in the notation in problem 3 is often called the representative of a coset. Hypothesize why we call the $g \in G$ the *representative* of $g + H$.

5. Identify any other left cosets of H in $\mathbb{Z}$.

6. Find the left cosets of $\langle 3 \rangle$ in $U(8)$.

7. Find $[U(8) : \langle 3 \rangle]$.

8. Prove the following: Let H be a subgroup of a group G and suppose that g_1 and g_2 are elements of G. Show $g_1 H = g_2 H$ if and only if $g_2 \in g_1 H$.