

## Suggested HW 9

1. Consider the mapping from  $G \oplus H$  to  $G$  given by  $(g, h) \rightarrow g$ .
  - (a) Determine if the map is a homomorphism.
  - (b) If the map is a homomorphism, determine the kernel.
2. Suppose that  $\phi$  is a homomorphism from  $\mathbb{Z}_{30}$  to  $\mathbb{Z}_{30}$  and  $\text{Ker}\phi = \{1, 10, 20\}$ . If  $\phi(22) = 8$ , determine all elements that map to 8.
3. If  $\phi$  is a homomorphism from  $\mathbb{Z}_{30}$  onto a group of order 5, find  $\text{Ker}\phi$ .
4. Consider the map  $\phi : \mathbb{Z} \oplus \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $\phi(a, b) = a - b$ .
  - (a) Determine if  $\phi$  is a homomorphism.
  - (b) If  $\phi$  is a homomorphism, determine the kernel.
  - (c) Describe the set  $\phi^{-1}(3)$ .
5. Let  $\mathbb{Z}[x]$  be the group of polynomials in  $x$  with integer coefficients under addition. Prove the mapping from  $\mathbb{Z}[x]$  to  $\mathbb{Z}$  given by  $f(x) \rightarrow f(3)$  is a homomorphism and give a geometric description of the kernel of the homomorphism.