TMath 402 Section 7

WrittenHW 7

1. [3] Let k be a fixed integer and define $H = \{0, \pm k, \pm 2k, \pm 3k, ...\}$.
   (a) Find all the left cosets of H in $\mathbb{Z}$.
   (b) Find all the right cosets of H in $\mathbb{Z}$.
   (c) Find $[\mathbb{Z} : H]$.
2. [3] Let $\text{o}(a) = 30$. Find all the left cosets of $\langle a^8 \rangle$ in $\langle a \rangle$.
3. [3] Let $\mathbb{C}^*$ be the group of nonzero complex numbers with multiplication.
   Let $H = \{a + bi | a^2 + b^2 = 1\}$. Give a geometric description of the cosets of H.
4. [4] Let the order of G be 15. If G has only one group of order 3 and only one group of order 5, prove that G is cyclic.
5. [2] Suppose a group contains elements of orders 1 through 9. What is the minimum possible order of the group? Justify your answer.

HW7 Writing Focus

1. [5] Let p and q be prime numbers. Let a and b be nonidentity elements of different orders in a group G and $|G| = pq$. Prove that the only subgroup of G containing a and b is G itself.
2. [5] Suppose that G is an Abelian group with an odd number of elements. Show the product of all the elements of G is the identity.
3. [5] Prove that 3, 5, and 7 are the only three consecutive odd integers that are prime.