WrittenHW 5

1. Consider the set $S = \{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10. Does S have unity? Justify your answer.
2. Let n be and integer and consider the ring $\mathbb{Z}_n$. For each of the properties below determine if the statements is *always* true in $\mathbb{Z}_n$. If true, give a brief explanation of why. If false, provide a counterexample.
   (a) If $a^2 = a$, then $a = 0$ or $a = 1$.
   (b) If $ab = 0$, then $a = 0$ or $b = 0$
   (c) If $ab = ac$ and $a \neq 0$, then $b = c$.
3. Find the subring lattice for $\mathbb{Z}_{12}$.
4. Is $\mathbb{Z}_6$ a subring of $\mathbb{Z}_{12}$? Justify your answer.

HW5 Writing Focus

1. Let m and n be positive integers and let $k = lcm(m, n)$. Prove that $m\mathbb{Z} \cap n\mathbb{Z} = k\mathbb{Z}$

WrittenHW 5

1. For each of the following determine if the set and binary operators form a field.
   (a) $\mathbb{Z}$ with standard addition and multiplication.
   (b) The Gaussian integers $\mathbb{Z}[i] = \{a + bi | a, b, \in \mathbb{Z}\}$ with standard addition and multiplication.
   (c) The ring $\mathbb{Z}[x]$ of polynomials with integer coefficients and polynomial addition and polynomial multiplication.
   (d) $\mathbb{Z}_p$ where p be prime using addition modulo p and multiplication modulo p.
   (e) The ring $\mathbb{Z}[\sqrt{3}] = \{a + b\sqrt{3} | a, b \in \mathbb{Z}\}$ with standard addition and multiplication.
2. Find the zero-divisors of $\mathbb{Z}_{20}$.

HW5 Writing Focus

1. Prove the set of nilpotent elements of a commutative ring form a subring.
   Define x to be nilpotent if there exists and integer n so that $x^n = 0$.
2. Find a necessary and sufficient condition on n and k so that k is a zero-divisor in $\mathbb{Z}_n$. Prove your statement.
3. Let R be a ring with m elements. Prove that the characteristic of R divides m.