TMath 402 Section 4

WrittenHW 4

1. ($\approx$ #8) Let a be an element of a group G and let $o(a) = 15$. Compute the orders of the following elements of G
   (a) a^3, a^6, a^9, a^{12}
   (b) a^5, a^{10}
   (c) a^2, a^4, a^8, a^{14}
2. ($\approx$ #10) Let $G = \langle a \rangle$ and let $o(a) = 24$. Find all generators for the subgroups of order 8.
3. ($\approx$ #14*) Suppose that a sucylic group G has exactly three subgroups: G itself, $\{e\}$, and a subgroup of order 7. Find $|G|$ and prove your conclusion.
4. ($\approx$ #32) Determine the subgroup lattice for $\mathbb{Z}_{12}$.
5. ($\approx$ #60*) Suppose that $o(a) = n$. Find a necessary and sufficient condition on r and s so that $\langle a^r \rangle \subseteq \langle a^s \rangle$.