WrittenHW 3

1. ($\approx$ #8) Consider the set $S = \{5, 15, 25, 35\}$. Does S form a group under multiplication modulo 40? If so, what is the identify element and describe any relationships between S and $U(8)$.
2. ($\approx$ #16) In a group, prove that $(ab)^{-1} = b^{-1}a^{-1}$. Draw an analogy between the statement $(ab)^{-1} = b^{-1}a^{-1}$ and the act of putting on and taking off your socks and shoes. Find an example that shows that it is possible to have $(ab)^{-2} \neq b^{-2}a^{-2}$. Find a distinct nonidentity elements A and B from a non-Abelian group with the property that $(ab)^{-1} = a^{-1}b^{-1}$.
3. ($\approx$ #26) Let G be a group and $a, b \in G$. Prove $(ab)^2 = a^2b^2$ if and only if $ab = ba$.
4. ($\approx$ #32) In D_n, let $r = R_{360/n}$, and let f be any reflection. Show that $frf = r^{-1}$ then use this relation to write the following elements in the form r^i or r^if where $0 \leq i < n$.
   (a) In D_4, $fr^{-2}fr^5$
   (b) In D_5, $r^{-3}fr^4fr^{-2}$
   (c) In D_6, $fr^5fr^{-2}f$

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1. ($\approx$ #14*) If H and K are subgroups of G, is $H \cap K$ is a subgroup of G? Prove your conclusion or find a counterexample.
2. ($\approx$ #22) Is the center of a group Abelian? Prove your conclusion or find a counterexample.
3. ($\approx$ #28) Consider the elements $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ from $SL(2, \mathbb{R})$.
   Using textbook notation: Find $|A|$, $|B|$ and $|AB|$.
   Alternative notation: Find $\text{o}(A)$, $\text{o}(B)$ and $\text{o}(AB)$.
4. ($\approx$ #46*) Let G be a group of functions from $\mathbb{R}$ to $\mathbb{R}^*$, where the operations of G is multiplication of functions. Let $H = \{f \in G | f(1) = 1\}$. Is H a subgroup of G? Prove your conclusion or find a counterexample.
5. ($\approx$ #50acd) Find the smallest subgroup of $\mathbb{Z}$ containing:
   (a) 8 and 14
   (b) 6 and 15
   (c) m and n