Midterm

TMath 402

Winter 2016

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. [3] (§0) Suppose a, b, $c \in \mathbb{Z}$ such that $a|c$ and $b|c$, then $(ab)|c$.

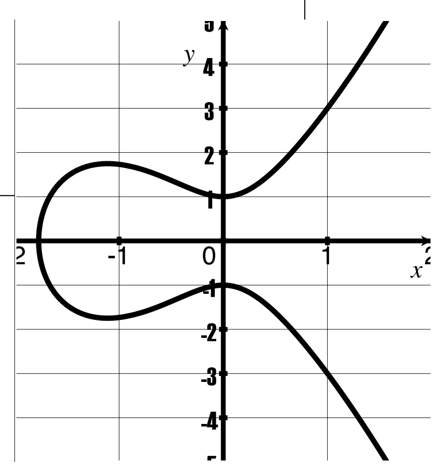
2. [3] (§1) Let a, b, and c be elements of D_4 where D_4 is the group of order 8. If $ab = bc$ then $a = c$.

3. [3] (§0) Define a relation R on the set $\{0, a, b, c\}$ so that $0R0$, $0Ra$, aRa, $aR0$, bRb, $bR0$, and cRc. The relation R defines an equivalence relation.

4. [3] (§4) Let G be a group and $G = \langle x \rangle$ If g, $h \in G$, then there exists $m \in \mathbb{Z}$ such that $x^m g = h$.

5. [9] (§2) For each of the sets and operations below, fill in the entries with yes or no. If no, briefly explain why. If yes, briefly describe the process you used to reach that anwer.

Sets S & Operator $\star$	Forms a Group	Abelian Group
a) $\{1, i, j, k, -1, -i, -j, -k\}$ $\begin{array}{ccccccccc} \star & 1 & i & j & k & -1 & -i & -j & -k \\ 1 & 1 & i & j & k & -1 & -i & -j & -k \\ i & i & -1 & -k & j & -i & 1 & k & -j \\ j & j & k & -1 & -i & -j & -k & 1 & i \\ k & k & -j & i & -1 & -k & j & -i & 1 \\ -1 & -1 & -i & -j & -k & 1 & i & j & k \\ -i & -i & 1 & k & -j & i & -1 & -k & j \\ -j & -j & -k & 1 & i & j & k & -1 & -i \\ -k & -k & j & -i & 1 & k & -j & i & -1 \end{array}$		
b) $\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle\vert ab \neq 0 \,\&\, a, b \in \mathbb{R} \right\}$ $\star$ is matrix multiplication		
b) $\{(x, y) \in \mathbb{R}^2 \vert y^2 = 3x^3 + 5x^2 + 1\}$ (graph shown to the right) $\star$ Let P, $Q \in S$. If $P \neq Q$ draw the line between P & Q. The line will intersect with S at a third point, call it C. Then $P \star Q = C$ If $P = Q$, draw the line tangent to the curve. The tangent line with intersect S at a second point, call it C. Then $P \star Q = C$		



Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
(If you use a calculator, be sure to tell me.)

6. (§3) Let G be a group and $g \in G$.

   (a) [2] Which is larger: the order of G or the order of g. Justify your answer.

   (b) [3] When is $|G| = o(g)$? (In Gallian's notation: $|G| = |g|$.) Justify your answer.

7. [6] (§3) Let G be an Abelian group with identity e and let m be a fixed integer. Show the set of all elements in G that are solutions to $x^m = e$ is a subgroup of G. In symbols show that $\{g \in G | g^m = e\} \leq G$.

8. (§4) Let $G = \mathbb{Z}_{338}$ under addition modulo 60.

   (a) [1] Find $|G|$.

   (b) [3] List three elements of order 26 in G.

   (c) [4] Write down the subgroup lattice for $\mathbb{Z}_{338}$

Writing Focus

This section is to be taken home, completed, and turned in by 12:50 Tuesday Feb 9th. You may discuss this problem with anyone else from the class. You may not consult anyone or any resource that is not affiliated with the class. That is, you may use the textbook and your classmates as a resource but you may not use other textbooks, the internet, or tutors.

Your answer may be typed or written. Answers will be evaluated as a completed, polished *proof*. Assume the audience are your peers in the class but write as you would for a published proof. An approximate rubric is given below.

- [1] correctness
- [2] generality/abstraction
- [1] notation
- [2] logic
- [2] style
- [2] TBD

9. [10] Let G be a finite group with more than one element. Show that G has an element of prime order.