Midterm

TMath 402

Winter 2014

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. [4] (§0 #7) If a and b are positive integers, then $ab = \operatorname{lcm}(a, b) \cdot \gcd(a, b)$.

2. [4] (§1 #11) Let a, b, and c be elements in the group D_4. If $ab = bc$ then $a = c$.

3. [4] (§3) Let G be a group and H a subset of G such that H is closed under the group operator. Then H is a subgroup of G.

4. [4] (§2) If $(ab)^2 = a^2b^2$ in a group G, then G is abelian.

5. [9] (§2) For each of the sets and operations below, fill in the entries with yes or no. If no, briefly explain why. If yes, briefly describe the process you used to reach that anwer.

Sets S & Operator $\star$	Forms a Group	Abelian Group	
a) $\{e, a, b, c\}$ $\begin{array}{ccccc} \star & e & a & b & c \\ e & e & a & b & c \\ a & a & e & c & b \\ b & b & c & e & a \\ c & c & b & a & e \end{array}$			
b) $\mathbb{Z}_8$ $\star$ standard multiplication			
c) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle\vert ad - bc \neq 0 \& a, b, c, d \in \mathbb{R} \right\}$ $\star$ is matrix multiplication			

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
(If you use a calculator, be sure to tell me.)

6. [3] Describe one application in the world that makes use of one of the mathematical structures discussed thus far in class.

7. [8] (§3) Let G be a finite group and consider the Cayley table for G. (i.e. the table that shows all possible pairs of binary operators of G). Show that every element of G occurs precisely once in each row of the table.

8. Consider the set $S = \{1, 2, 3, ...n - 2, n - 1\}$ where n is a positive integer.

   (a) [2] (§2 #35) When does S form a group under multiplication?

   (b) [6] (§2 & 0) Prove your assertion.

9. (§4) Let $G = \mathbb{Z}_{60}$ under addition modulo 60.

(a) [2] Find $|G|$.

(b) [3] Calculate the order of the element 49 in G.

(c) [3] List four elements of order 15 in G.

(d) [2] Find a subgroup of G that has order 5.

(e) [6] Write down the subgroup lattice for $\mathbb{Z}_{60}$