Reading Quiz §6 & 8

Key

1. Recall that the Klein 4 group V has a Cayley table shown to the right. Recall also that $\mathbb{Z}_2$ forms a group under addition modulo 2.

V	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

(a) [2] Write down the elements in $\mathbb{Z}_2 \times V$.

$$\left\{ \begin{matrix} (0,e) & (1,e) \\ (0,a) & (1,a) \\ (0,b) & (1,b) \\ (0,c) & (1,c) \end{matrix} \right\}$$

(b) [1] Compute $(1,a) \star (0,b)$ in $\mathbb{Z}_2 \times V$.

$$(1,a)*(0,b) = (1 \oplus 0, a \cdot b) \quad \Big\} .5$$
$$= (1,c) \quad \Big\} .5$$

2. [2] Define the alternating group on 3 elements A_3, and write down explicitly a group element that is not the identity.

+1 { A_3 consists of all permutations in S_3 that are even.

Recall, a permutation is even if x can be written as a product of an even # of transpositions.

+1 ex $\underbrace{(1\,2)(2\,3)}_{\text{only 2 transpositions}} = (1\,2\,3)$