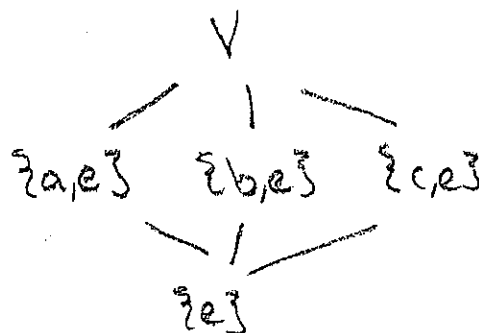


Reading Quiz §5

Key

1. [1] Identify a subgroup in the Klein 4 group V . Recall that the Cayley table for V is:

V	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e



any of the 3 groups on the right would suffice.

2. [3] Consider the group of integers modulo 8 with modular addition, (\mathbb{Z}_8, \oplus) . If possible, find subgroups of \mathbb{Z}_8 of order:

(a) 2

$$\langle 4 \rangle = \{4, 0\}$$

(b) 3

There are no subgroups of order 3 in \mathbb{Z}_8 .

One of the theorems in the text said,

If H is a subgroup of G , then $|H| \mid |G|$.

But $3 \nmid 8$.

3. [1] Describe the Quaternion Group briefly. For example, what is its order? Is it a subgroup of anything? Write down something that you know about it.

The Quaternions can be thought of as a subset of $GL_2(\mathbb{C})$:

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \right\}$$

$$\{ e, -e, i, -i, j, -j, k, -k \}$$

The binary operation is matrix multiplication or

$$i^2 = j^2 = k^2 = -e$$

and

$$\begin{matrix} i & j & k \\ j & k & i \\ k & i & j \end{matrix}$$

can be used to determine products between the elements.