Reading Quiz §3

1. TRUE/FALSE: Identify the statement as True in each of the following cases if the statement is *always* true and provide brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) [2] Every group has two distinct identity elements.

   (b) [2] The statement: *If $(G, \star)$ is a group and x is any element of G, then x has only one inverse in G.*
   Can be proved in *only* the following way: *Let e be the identity element in G. Assume y and z are both inverses to x. Then we know:*
   $y \star x = x \star y = e$ *and* $z \star x = x \star z = e$ *($\Diamond$).*
   We want to show that $y = z$.
   Certainly $e = e$, then by ($\Diamond$) we know $x \star y = x \star z$ ($\heartsuit$).
   If we multiply both sides of ($\heartsuit$) on the right by y we have: $y \star (x \star y) = y \star (x \star z)$.
   Since $\star$ is associative we can rewrite the above to $(y \star x) \star y = (y \star x) \star z$ which simplifies to $e \star y = e \star z$ by ($\Diamond$). Thus $y = z$.

2. [1] Consider Theorem 3.7: Let G be a set and $\star$ an associative binary operation on G. Assume that there is an element $e \in G$ such that $x \star e = x$ for all $x \in G$, and assume tat for any $x \in G$ there exists and element y in G such that $x \star y = e$. Then $(G, \star)$ is a group.

   How is the above theorem different from the definition of a group given in §2?