Reading Quiz §2

Key

Section 2 of Saracino's Abstract Algebra book gives the following definition:
Suppose that:

1. G is a set and $\star$ is a binary operation on G,
2. $\star$ is associative,
3. there is an element e in G such that $x \star e = e \star x = x$ for all x in G, and
4. for each element $x \in G$, there is an element $y \in G$ such that $x \star y = y \star x = e$.

1. [1] Provide an example set and binary operator that forms a group.

   ($\mathbb{R}$, addition)
   ($\mathbb{Z}$, addition)
   ($\mathbb{Q}$, multiplication)

   Symmetries of the square with composition
   n^{th} roots of unity with multiplication
   2x2 matrices under addition
   General linear group of 2x2 matrices under multiplication

2. [1] Identify the identity e in the example group you provided above in problem 1.

   zero
   zero
   1

   no movement
   1
   $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
   $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. [1] What property makes a group abelian?

   the elements commute.
   That is, if $x, y \in G$ for an abelian group G, then $x \cdot y = y \cdot x$.
   note: the sym. of square & general linear groups above are not abelian

4. [2] In the additive group of integers $\mathbb{Z}_{16}$ find:

   (a) 15+2 $\equiv 17 \mod 16$
   $\equiv 1 \mod 16$

   (b) 15+18 $\equiv 33 \mod 16$
   $\equiv 1 \mod 16$