Reading Quiz §13

Key

1. [1] How many Isomorphism Theorems are there?

   three

2. [3] Find the kernel of each of the following homomorphisms:

$$D_4 \xrightarrow{\phi} V \qquad\qquad (\mathbb{Z}, +) \xrightarrow{x} (\mathbb{Z}_6, \oplus)$$
$$e \mapsto e \qquad\qquad\qquad x \mapsto x \mod 6$$
$$r \mapsto a$$
$$r^2 \mapsto e$$
$$r^3 \mapsto a \qquad\qquad \langle 6 \rangle = \{0, 6, 12, 18, \dots$$
$$f \mapsto b \qquad\qquad\qquad\qquad -6, -12, -18, \dots \}$$
$$rf \mapsto c$$
$$r^2 f \mapsto b$$
$$r^3 f \mapsto c$$

$$\text{Ker } \phi = \{e, r^2\}$$

3. [3] The factor group $D_4/\{e, r^2\}$ is isomorphic to a group of order 4. Prove the isomorphism (ideally with the first isomorphism and some work from above.

   Notice ϕ defined above is an onto homomorphism with $\text{Ker } \phi = \{e, r^2\}$. Thus we have by the first isomorphism Thm an isomorphism $\psi : D_4/\text{Ker } \phi \to V$. //

   $$D_4 \xrightarrow{\phi} V$$
   $$D_4/\text{Ker } \phi$$

4. [3] Write down the partial subgroup lattice of subgroups in D_4 that also contains $\{e, r^2\}$.

   Note $D_4/\{e, r^2\} \cong V$

   $$\langle a \rangle \quad \langle b \rangle \quad \langle c \rangle$$
   $$\langle e \rangle$$

   by the subgroup lattice correspondence then subgps in D_4 containing $\{e, r^2\}$:

   $$D_4$$
   $$\{r, r^3, e, r^2\} \quad \{f, r^2 f, e, r^2\} \quad \{rf, r^3 f, e, r^2\}$$
   $$\{e, r^2\}$$

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