Reading Quiz §12

Key

1. True/False: Determine whether each of the following is *always* true or false. If false, provide a counterexample. If true, provide a proof.

   Let G and H be groups and let K be a subgroup of G.
   Let $\phi : G \to H$ be a homomorphism.

   (a) [2] Then $\phi(x^3) = [\phi(x)]^3$.

   True $\phi(x \cdot x \cdot x) = \phi(x)\phi(x \cdot x)$ b/c ϕ is a homomorph.
   $= \phi(x)\phi(x)\phi(x)$ "
   $= [\phi(x)]^3$

   (b) [2] Then $\phi(K)$ is a subgroup of H.

   True Notice $e_K \in K$ so $\phi(K)$ is nonempty.
   Let $k, j \in K$ then $\phi(k)\phi(j) = \phi(kj)$ b/c ϕ is homomorph
   and $\phi(kj) \in \phi(K)$ so we have closu
   Let $k \in K$, then $\phi(k^{-1})$ is the inverse to $\phi(k)$
   b/c ϕ is a homomorphism $\therefore \phi(K) \leq H$

2. [6] Determine if the following maps are homomorphisms, monomorphisms, epimorphisms, and/or isomorphims.

   $(\mathbb{Z}_2, \oplus) \xrightarrow{\phi} (\mathbb{Z}_6, \oplus)$
   $0 \mapsto 0$
   $1 \mapsto 1$

   $\phi(1+1) \stackrel{?}{=} \phi(1) + \phi(1)$
   $\phi(0) \quad 1 + 1$
   $0 \quad 2$
   no not homomorphism

   $(\mathbb{Z}_6, \oplus) \xrightarrow{\psi} (\mathbb{Z}_6, \oplus)$
   $0 \mapsto 0$
   $1 \mapsto 1$
   $2 \mapsto 0$
   $3 \mapsto 1$
   $4 \mapsto 0$
   $5 \mapsto 1$

   $\psi(3+3) \stackrel{?}{=} \psi(3) + \psi(3)$ ie
   $\psi(0) \quad 1+1$
   $0 \neq 2$
   no not a homomorphism

   $(\mathrm{GL}(2, \mathbb{R}), \cdot) \xrightarrow{\chi} (\mathbb{R} \setminus \{0\}, \cdot)$
   $A \mapsto \det(A)$

   homomorph b/c
   The det map respects matrix multiplication
   $\det(A \cdot B) = \det(A)\det(B)$
   not one-to-one only b/c
   $\det \begin{pmatrix} r & 0 \\ 0 & 1 \end{pmatrix} = r$ for any $r \in \mathbb{R} - \{0\}$

   epimorphism