

# Reading Quiz §12

1. True/False: Determine whether each of the following is *always* true or false. If false, provide a counterexample. If true, provide a proof.

Let  $G$  and  $H$  be groups and let  $K$  be a subgroup of  $G$ .

Let  $\phi : G \rightarrow H$  be a homomorphism.

(a) [2] Then  $\phi(x^3) = [\phi(x)]^3$ .

(b) [2] Then  $\phi(K)$  is a subgroup of  $H$ .

2. [6] Determine if the following maps are homomorphisms, monomorphisms, epimorphisms, and/or isomorphisms.

$$(\mathbb{Z}_2, \oplus) \xrightarrow{\phi} (\mathbb{Z}_6, \oplus)$$

$$0 \mapsto 0$$

$$1 \mapsto 1$$

$$(\mathbb{Z}_6, \oplus) \xrightarrow{\psi} (\mathbb{Z}_6, \oplus)$$

$$0 \mapsto 0$$

$$1 \mapsto 1$$

$$2 \mapsto 0$$

$$3 \mapsto 1$$

$$4 \mapsto 0$$

$$5 \mapsto 1$$

$$(\mathrm{GL}(2, \mathbb{R}), \cdot) \xrightarrow{\chi} (\mathbb{R} \setminus \{0\}, \cdot)$$

$$A \mapsto \det(A)$$