Reading Quiz §10

Key

1. [2] Let H be a subgroup of a group G. Recall that the index of H in G is denoted $[G : H]$. Find the following:

$[\mathbb{Z}, < 2 >]$

$<2> = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$

$<2>+1 = \{\ldots -3, -1, 1, 3, 5, \ldots\}$

$\therefore$ # of right cosets $= [\mathbb{Z}, <2>] = 2$

$[\mathbb{Z}_{12}, < 4 >]$

$<4> = \{4, 8, 0\}$ $\quad <4>+2 = \{6, 10, 2\}$

$<4>+1 = \{5, 9, 1\}$ $\quad <4>+3 = \{7, 11, 3\}$

$\therefore$ # of right cosets $= [\mathbb{Z}_{12}, <4>] = 4$

2. Let G be a group and $y \in G$. Recall the centralizer of of y in G is:

$$Z(y) = \{x \in G | xy = yx\}$$

(a) [1] How is the centralizer of y different from the center of G?

Center: $Z(G) = \{x \in G \mid xz = zx \ \forall z \in G\}$

↳ all elements in G that commute with all others

centralizer of y

↳ all elements in G that commute with y

(b) [1] Find $Z(2)$ in the group $\mathbb{Z}_5$, under modular addition.

$\mathbb{Z}_5$ is abelian so all elements in $\mathbb{Z}_5$ commute with $\bar{2}$

$\Rightarrow Z(\bar{2}) = \mathbb{Z}_5$

3. [1] Explain, define, or provide an example of a conjugacy class. Let G be a group & $g \in G$

conjugacy class of $g = \{xgx^{-1} \in G \mid x \in G\}$

For example: the conjugacy class of $\bar{2}$ in $\mathbb{Z}_5$ is $\{\bar{2}\}$ [Primarily b/c $\mathbb{Z}_5$ is abelian]

Another example: In S_3 the conjugacy class of $(1,2,3)$ is $\{(1,2,3), (132)\}$

$(132)(231) = (1)(2)(3)$

$(12)(123)(12) = (132)$

$(132)(123)(231) = (123)$

$(13)(123)(13) = (132)$

$(23)(123)(23) = (132)$

✓ (12)

✓ (13)

✓ (23)

(123)

✓ (132)

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