FINAL

TMath 402

Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. The relation R on the set $\mathbb{Z}$ defined below is an equivalence relation.

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} | 5 \text{ divides } a - b\}.$$

2. The set of all pairs (x, y) of real numbers such that $y \neq 0$, under the operation $(x, y) \star (z, w) = (x + z, yw)$.

3. If G is a group in which $(ab)^2 = a^2b^2$ for all $a \in G$ and $b \in G$, then G is abelian.

4. If H is a normal subgroup of a group G, then $ghg^{-1} = h$ for all $h \in H$ and $g \in G$.

5. If H and K are subgroups of G, $H \cong K$, then $G/H \cong G/K$.

6. The group $\mathbb{Z}_{10} \times \mathbb{Z}_{85}$ is cyclic.

7. All rings have an abelian group structure.

8. The element $(2, 3)$ is a zero divisor in the ring $\mathbb{Z}_4 \times \mathbb{Z}_6$.

9. For each of the algebraic structures, list *all* properties that it is guaranteed to satisfy:

Algebraic Structures	Properties
a) vector space	A) the set os closed under addition
b) group (assume binary operation is multiplicative)	B) the set is closed under multiplication
c) quotient/factor group (assume binary operation is multiplicative)	C) there is a 0 (additive identity)
d) cyclic group (assume binary operation is multiplicative)	D) there is a 1 (multiplicative identity)
e) normal subgroup (assume binary operation is multiplicative)	E) multiplication is commutative
f) ring	F) addition is commutative
g) division ring	G) cancellation laws hold (for all binary operations)

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
(If you use a calculator, be sure to tell me.)

10. For a group G and an element of the group g, which is larger: the order of the group or the order of the element? When are they equal?

11. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 5 & 6 & 4 & 7 & 1 \end{pmatrix}$ from S_8 as:

(a) a product of disjoint cycles;

(b) a product of transpositions.

(c) Compute σ^4.

(d) Find the order of σ.

(e) Write down σ^{-1}.

12. Classify each of the following maps as either: not a homomorphism, homomorphism, monomorphism, epimorphism, isomorphism, or automorphism. Prove your classification.

$$\begin{aligned} \mathbb{Z}_6 &\to \mathbb{Z}_{18} \\ x &\mapsto 3x+5 \end{aligned} \qquad\qquad \begin{aligned} (\mathbb{Q},+) &\xrightarrow{\chi} (\mathbb{Q},+) \\ \tfrac{m}{n} &\mapsto \tfrac{m^2}{n^2+1} \end{aligned}$$

13. Groups D_4 and Q_8 have 8 elements. Show that they are not isomorphic.

14. Suppose that H is a subgroup of G. Show that the intersection of all conjugate subgroups of H, $\bigcap\limits_{x \in G} xHx^{-1}$, forms a normal subgroup of G.

15. Let $G = S_4$ and H be something non trivial that makes sense. Define $\phi : G \to H$...

(a) Create the subgroup lattice of G.

(b) Find $\ker(\phi)$.

(c) The factor group $G/\ker(\phi)$ is isomorphic to a familiar group talked about in this course. Find out what it is and then build an isomorphism.

(d) Draw the subgroup lattice of $G/\ker(\phi)$.