Take Home Midterm

TMath 402

Autumn 2020

This section is to be taken home, completed, and turned in through Canvas by 10:10 Tuesday Nov 3rd. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

1. Dr. Vanderpool provides you with Cayley table for a group W given below:

$\star$	z	x	y	w	r	s	t	u
z	z	x	y	w	r	s	t	u
x	x	r	u	y	s	z	w	t
y	y	w	r	s	t	u	z	x
w	w	t	x	r	u	y	s	z
r	r	s	t	u	z	x	y	w
s	s	z	w	t	x	r	u	y
t	t	u	z	x	y	w	r	s
u	u	y	s	z	w	t	x	r

   (a) [1] Find a generating set for W.

   (b) [2] Find a minimal generating set for W. Prove that the set you provide is minimal.

   (c) [3] Draw a Cayley graph for W based on the elements you find in part (b).

2. Consider Dr. Vanderpool's proof written for the theorem below.

   (a) [2] Identify any logical errors.

   (b) [2] Provide concrete suggestions for how the proof can be improved.

Theorem 1 (3.23)**.** *Let G be a group and let $g_1, g_2, \ldots g_n \in G$. If $x \in \langle g_1, g_2, ...g_n\rangle$, then $\langle g_1, g_2, ...g_n\rangle = \langle g_1, g_2, ...g_n, x\rangle$.*

Proof. Recall, by definition $\langle g_1, g_2, ...g_n\rangle = \{y_i y_2 y_3 \cdots y_m | \forall i, y_i \in \{g_1, g_2, ...g_n, g_1^{-1}, g_2^{-1}, ...g_n^{-1}\}\}$. Recall also from Chapter 2, that $\langle g_1, g_2, ...g_n\rangle \leq G$.

Since $x \in \langle g_1, g_2, ...g_n\rangle$, we know $x = g_i$ for some i. Thus,

$$\begin{aligned} \langle g_1, g_2, ...g_n, x\rangle &= \langle g_1, g_2, ...g_n, g_i\rangle \\ &= \langle g_1, g_2, ...g_n, \rangle \end{aligned}$$

where the last equality holds because sets are unordered and can ignore duplications.

$\square$