Vanderpool

TMath 402

Autumn 2020

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. [3] (§2.1) Every word in $\text{Spin}_{3\times 3}$ is its own inverse.

2. [3] (§2.2) Let $*$ be a binary operator on a set A. If $*$ is associative and commutative then $(a*b)*(a*b) = (a*a)*(b*b)$ for all a, $b \in A$.

3. [3] (§2.3) Let a, b, and c be elements of D_4 where D_4 is the group of order 8. If $ab = bc$ then $a = c$.

4. [3] (§3.1) If H is an abelian subgroup of G, then G is abelian.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

5. [6] (§2.3) For each of the sets and operations below, fill in the entries with yes or no. If no, briefly explain why. If yes, briefly describe the process you used to reach that anwer.

Sets S & Operator $\star$	Is a Binary Operator?	Is there an Identity? leave blank if not a binary operator
a) $\mathbb{R}$ $\star$ is defined as $f(x) = 2x$		
b) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle\vert ad - bc \neq 0 \,\&\, a, b, c, d \in \mathbb{R} \right\}$ $\star$ is matrix multiplication		

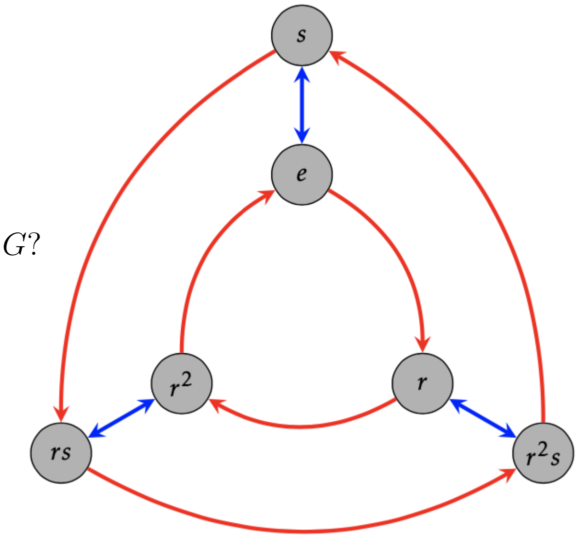
6. Dr. Vanderpool provides you with the same Cayley Table for group W. Use it to answer the questions below.

(a) [2] (§2.5) Identify the identity. Explain how you know your answer is the identity.

$\star$	z	x	y	w	r	s	t	u
z	z	x	y	w	r	s	t	u
x	x	r	u	y	s	z	w	t
y	y	w	r	s	t	u	z	x
w	w	t	x	r	u	y	s	z
r	r	s	t	u	z	x	y	w
s	s	z	w	t	x	r	u	y
t	t	u	z	x	y	w	r	s
u	u	y	s	z	w	t	x	r

(b) [2] (§2.5) Find a pair of non-identity elements that commute with each other.

7. Let G be the group whose Cayley Diagram is on the right.



(a) [1] (§3.1) How many elements are in the group G?

(b) [2] (§3.1) Which arrows correspond to which generators in our Cayley Diagram of G?

(c) [1] (2.4) Describe r^{-1} in terms of the generators on the Cayley Diagram

(d) [3] (§2.6) Is $\{r\}$ a generating set for G? Justify your answer.

(e) [3] (§3.2) Use the Cayley Diagram to build a subgroup lattice for G.

8. [8] Choose *ONE* of the following theorems to prove. Clearly identify which of the two you are proving and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

Theorem 1. *Let G be a group with identity element e. If $x^2 = e$ for all $x \in G$, then G is abelian.*

Theorem 2 (3.24)**.** *If G is a group such that $H, K \leq G$, then $H \cap K \leq G$.*