Vanderpool

TMath 402

Autumn 2020

True/False: If the statement is false, give a counterexample.
If the statement is *always* true, give a brief explanation of why it is (not *just* an example!).

1. [3] (§2.6) Let G_1 and G_2 be groups such that $G_1 \cong G_2$. Then the Caley Diagram of G_1 matches the Caley Diagram of G_2.

2. [3] (§3.3) Consider the integers $\mathbb{Z}$ under addition. The map $\theta : \mathbb{Z} \to \mathbb{Z}$ defined by $\theta(x) = 9x$ satisfies the homomorphic property.

3. [3] (§3.1) If H is an abelian subgroup of G, then G is abelian.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

4. [6] (Chapter 2) Identify three non-isomorphic groups of order 8 (you can use the same notation for the groups we used in class). You must explain why each group is not isomorphic to the others.

5. (§2.4 & 8.1) For each question below, find a set and a binary operator(s) that satisfy the criteria, if one exists. If not set and binary operator(s) exist, explain why.

   (a) [3] a cyclic, but non-abelian group

   (b) [3] a ring, but not commutative

   (c) [3] an integral domain, but not a field

6. Let $G = \langle a \rangle$ be a group generated by a group of order 28.

   (a) [2] (§4.1) Find all the elements that generate G.

   (b) [2] (§4.1) Find an element in G that has order 4.

   (c) [4] (§3.2 & 4.1) Draw the subgroup lattice for G.

7. [8] Choose *ONE* of the following theorems to prove. Clearly identify which of the two you are proving and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

Theorem 1. *Let* $\phi : G_1 \to G_2$ *be a function between two groups that satisfies the homomorphic property. The kernel of* ϕ *is defined:*

$$K_\phi = \{g \in G_1 | \phi(g) = e_2\}$$

where e_1 *and* e_2 *are the identities of* G_1 *and* G_2 *respectively. Prove that* $K_\phi = \{e_1\}$ *if and only if* ϕ *is one-to-one.*

Theorem 2. *Let* G *be a group with identity element* e*. If* $x^2 = e$ *for all* $x \in G$*, then* G *is abelian.*