Exam 3

TMath 344

Spring 2019

Show all your work. Reasonable supporting work must be shown to earn credit.

1. [8] Let A and B be points on the half plane model of hyperbolic geometry. Let ϕ be a fold. Determine if the following make sense/*could* be true and be sure to justify your answer:

   (a) $A = B$

   (b) $B = (3, e^t)$ where $-\infty < t < \infty$

   (c) $A = \ln(6)$

   (d) $\phi(A) = A$

2. [4] (§7.1) Critique the following theorem and proof.

   Theorem 1. *Let $l \parallel m$ and ϕ a fold so that $\phi(l) = m$. Let n be the crease of ϕ. Then $l \parallel n \parallel m$.*

   Proof. Assume not, and let $P = l \cap n$. Then $\phi(P) = P \in l$ since $P \in n$. Thus $\phi(P) \notin m$, a contradiction. This completes the proof. □

Theorem Playfaire's. *Given a line l, and a point $A \notin l$, there is a unique line m through A and parallel to l.*

Recall a postulate is an assumed fact. Recall also that the parallel postulate (also known as Euclid's 5th postulate) is the statement "If two parallel lines are cut by a transversal, then alternate interior angles are congruent." Since mathematicians spent a lot of time trying to prove the parallel postulate followed from the previous postulates, there are many statements that are equivalent to the parallel postulate. For example, Playfaire's Theorem is true if and only if the parallel postulate is true.

3. [2] Provide another statement that is equivalent to the parallel postulate (that is not Playfaire's Theorem).

4. Consider how Playfaire's Theorem (and thus the parallel postulate) could *not* be true and examine the ramifications. In particular, describe the what geometry is created if:

   (a) [2] there is no line through A and parallel to l.

   (b) [2] there are 2 distinct lines through A & parallel to l.

   (c) [1] there are 3 distinct lines through A & parallel to l.

5. [4] (2Dpostulate Activity) Spherical geometry fails to satisfy the first Postulate ("Any two points determine a unique line."). Explain

   (a) how spherical geometry fails the first postulate, and

   (b) how the postulate can be modified to resolve this problem.

6. Consider the unit sphere in $\mathbb{R}^3$ model for spherical geometry. Let $A = (\frac{1}{3}, \frac{2}{3}, \frac{-2}{3})$ and $B = (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$

   (a) [1] Find the coordinates for $\dot{A}$

   (b) [3] (§8.1 #7) Find an equation for $\overleftrightarrow{AB}$

7. Consider the line l in the half plane model of hyperbolic geometry defined by $x^2+y^2=1$

   (a) [3] (2D HyperbolicActivity #1) Find a family (more than two!) of lines that are parallel to l.

   (b) [3] (§7.4 #27) Let $A = (\frac{-1}{2}, \frac{7}{25})$ and $B = (\frac{-1}{2}, \frac{10}{3})$. Find the point $C = l \cap \overleftrightarrow{AB}$.

8. [7] (5/28 Lecture) Find the distance between points $A = (\frac{1}{25}, \frac{7}{25})$ and $B = (\frac{2}{5}, \frac{4}{5})$ in the half plane model of hyperbolic geometry.

Recall lines on the unit sphere model of spherical geometry are intersections of the sphere (defined by $x^2 + y^2 + z^2 = 1$) with planes in $\mathbb{R}^3$ through the origin (defined by $ax + by + cz = 0$ where $(a, b, c) \neq (0, 0, 0)$). For each line in the unit sphere there is thus exactly two (antipodal) points $\pm(a, b, c)$ in the sphere whose coordinates are coefficients for the line.

Let us create a new geometry model called Projective Geometry and define "point" to mean pairs of antipodal points in the unit sphere. Define "line" as the set of points that are both on the sphere and on a plane in $\mathbb{R}^3$ that passes through the origin.

9. [2] Does the Projective Geometry satisfy Postulate 1? Justify your answer.

10. [2] Does the Projective Geometry satisfy Postulate 3? Justify your answer.

11. [6] Let l and m be two lines in hyperbolic space with a common perpendicular. Prove that l and m are ultra parallel.