Exam 2

TMath 344

Spring 2019

Key

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Show all your work. Reasonable supporting work must be shown to earn credit.

1. [8] Let A and B be points with coordinates a and b respectively. Let l and m be lines and ϕ a fold. Determine if the following make sense/*could* be true and be sure to justify your answer:

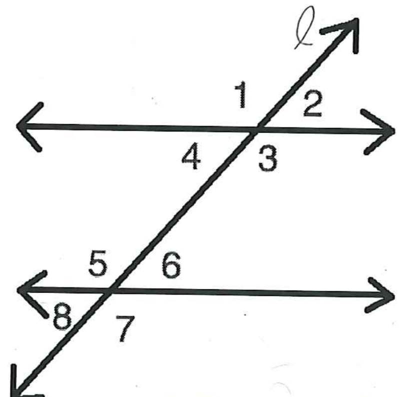
   (a) $A < B$ (+1) does not make sense
   (+1) [A and B are points which have no magnitudes for comparison

   (b) $l(A) = B$ (+1) [does not make sense
   (+1) [l is a line - I don't know what a line next to a point is suppose to mean.

   (c) $\phi(A) \in l$
   (+1) makes sense
   (+1) [A fold sends point A to another point which could be in line l

   (d) $\phi = A$
   (+1) does not make sense
   (+1) ϕ is a fold and A a point. Dif. objects cannot be equal.

2. Use the figure on the right to answer the following.



   (a) [1] Identify a transversal line.
   (+1) The diagonal one : l

   (b) [2] Identify a pair of angles in the figure that are non-alternating interior angles.
   (+1) (+1)
   4 & 5 work   3 & 6 work

   (c) [2] Give a condition on the angles that will guarantee two of the lines shown are parallel.
   If angle measure of 4 = angle measure of 6
   If angle measure of 5 = angle measure of 3
   If angle measure of 2 = angle measure of 6
   If angle measure of 2 = angle measure of 8

   +.5 measures are same
   +.5 each angle / pair
   +.5 incident w/ both lines

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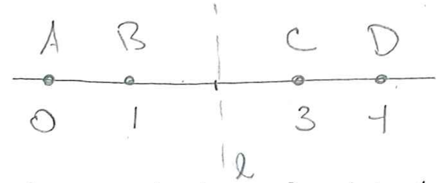

3. Fold a line and use a ruler to mark points A, B, C, and D with coordinates 0, 1, 3 and 4 units respectively.

(a) [3] (Practice 4.12) Determine if there exists a fold ϕ such that $\phi(A) = D$ and $\phi(B) = C$. If ϕ exists, define it clearly. If ϕ does not exist, explain why not.

(+.5) Start

Yes ? (+.1) (+1.5)

Let ℓ be the $\perp$ bisector of $\overline{BC}$.

Notice ℓ is also the $\perp$ bisector of $\overline{AD}$.

Define ϕ as the fold over crease ℓ

... it may not be clear this was a 2D question

(b) [3] Determine if there exists a fold θ such that $\theta(A) = B$ and $\theta(B) = C$. If θ exists, define it clearly. If θ does not exist, explain why not.

(+.5) Start

No? (+1) (+1.5)

Recall that folds preserve distance so

$\overline{AB} = \overline{\phi(A)\phi(B)}$ but $\overline{AB} = 1$ and $\overline{\phi(A)\phi(B)} = 2$

So $\nexists$ fold that can send $\theta(A) = B$ and $\theta(B) = C$

(c) [4] (2D congruence Activity #4) Determine if there exists an isometry τ such that $\overline{AB}$ is translated to $\overline{CD}$. If τ exists, define it clearly. If τ does not exist, explain why not. Yes (+.5)

note: there are lots of ways to do this?

(+.5) Start

Translations are the results of two folds.] (+.5)

(+.5) [Let τ_1 be the fold over the $\perp$ bisector of AC.
Note that $\tau_1(A) = C$ but the coord of $\phi(B) < 3$ (+1) get segment to be right

(+.5) [Let τ_2 be the fold over the line thru C and $\perp$ to $\phi(A)D$.
Note that $\tau_2(C) = C$ b/c of choice of crease

(+.5) [Define $\tau = \tau_2 \circ \tau_1$. Note $\tau_2 \circ \tau_1(A) = \tau_2(C) = C$ and the coord of $\tau_2 \circ \tau_1(B)$ is $2\cdot 3 - (2\cdot 1.5 - 1) = 4$ so $\tau_2 \circ \tau_1(B) = D$

(d) [4] (HW5 5.3) For each fold or isometry above that exists, find the inverse. That is, if ϕ exists, find ϕ^{-1}; if θ exists, find θ^{-1}; and if τ exists, find τ^{-1}.

(+.5) Start

(+1.5) [Inverse to fold is itself $\Rightarrow$ $\phi^{-1} = \phi$ in (a)

(+2) [Inverse to $\tau = \tau_2 \circ \tau_1$ is $\tau^{-1} = \tau_1^{-1} \circ \tau_2^{-1}$

or $= \tau_1 \circ \tau_2$

(we have to reverse the order)

+1

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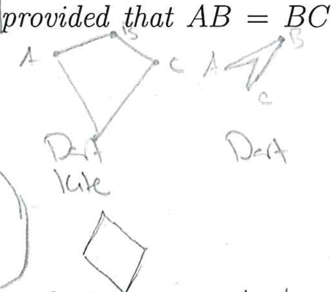
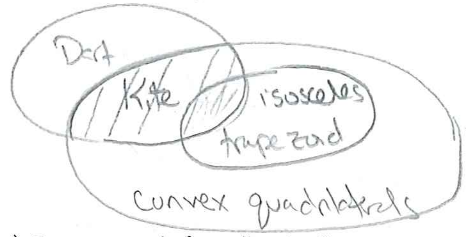
4. [5] (Suggested 6.29) Below are definitions for a few other special quadrilaterals. Explain the relation between them.

Definition 1. *A* $\boxed{\text{quadrilateral is convex}}$ *provided that both diagonals are contained in the interior of their respective angles. An* $\boxed{\text{isosceles trapezoid}}$ *is a trapezoid whose legs are congruent. A quadrilateral ABCD is a* $\boxed{\text{dart}}$ *provided that $AB = BC$ and $CD = AD$. A convex dart is called a* $\boxed{\text{kite}}$.





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5. (2DInteresting Activity) Let us redefine "point" to mean what you would usually call a line through the origin in $\mathbb{R}^3$.

(a) [2] How would "lines" be defined in this space?

Planes in $\mathbb{R}^3$ thru the origin +1

(notice planes thru the origin +.5 connect 2 lines thru the origin) start +.5

—or—

If using the hemisphere +.5 representation - great +.5 circles on the hemispheres connecting +1 two points

(b) [3] Do parallel lines in this geometry satisfy the parallel postulate? Recall the parallel postulate: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

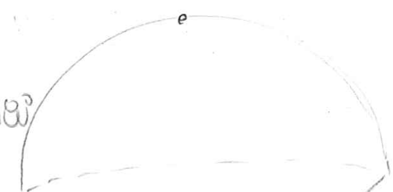
Nope.. The easiest way to see this is is to recall the // postulate is equivalent to Σ triangle angles $= 180°$ and then #7

+.5

logic +1

true +1

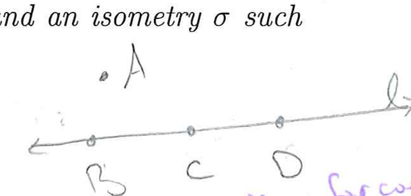
start +.5



6. [4] (HW5 5.15) Consider the conjecture below. If it is true, provide a proof, if it is false, provide a counter example.

start +.5

Conjecture 1. *Given four distinct points A, B, C, and D and an isometry σ such that $\sigma(B) = B$, $\sigma(C) = C$ and $\sigma(D) = D$, than $\sigma(A) = A$.*



False +1

Let B, C and D be colinear on a line l.

Let A be a point not on l.

Define σ as the fold with crease l.

Note $\sigma(A)$ is on a dif. half plane than A (postulate)

so $\sigma(A) \neq A$.

+2.5

+.5 looking for contrex

+1.5 def of line/fold/pos

+.5 is counter ex

if cite another thm that's correct +1

if start proof +.5

logic +1.5

+.5 style info

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7. [2] (5/7 Lecture) Identify a geometry that has the sum of angles in a triangle adding up to more than π radians.

start (1.5)

(+1.5) Oranges (according to Vittert)
or Spheres..

Note the triangle I drew has more than π radians

8. [5] (HW4 #4.59) Let A, B, and C be points, and let ϕ a fold such that $\phi(A) = C$, $\phi(C) = B$ and $\phi(B) = A$. Prove that $A = C$.

start (+1)
intro/style (+1)
on topic/fold (+1)
logic (+1.5)
notation (+.5)

Recall that if ϕ is a fold that $\phi(M) = N$ then $\phi(N) = M$.
Thus $\phi(C) = B \Rightarrow \phi(B) = C$.
But we were given $\phi(B) = A$.
Thus $A = \phi(B) = C$ or $A = C$. //

— or —

Recall that if ϕ is a fold that $\phi^2(M) = M$ for all points M.
Consider $\phi(\phi(C)) = \phi(B) = A$
The above recall implies $\phi(\phi(C)) = C$
Thus $A = \phi(\phi(C)) = C$ or $A = C$ //

— or —

9. [2] What concept did you study well and not see on the exam?

$$\frac{27 + 23}{50}$$

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