Key

Exam 1 | TMath 344 | Spring 2019

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Show all your work. Reasonable supporting work must be shown to earn credit.

1. [10] Let A, B, C, and D be points with coordinates a, b, c, and d respectively. Determine if the following make sense/*could* be true and be sure to justify your answer:

   (a) $A = a$ (+1) does not make sense

   (+1) [A is a point a is a number (a coordinate)
   not at all the same thing as the equal sign suggests.

   (b) $A = B$ (+1) makes sense

   (+1) Both A & B are points. A and B could be the same point.

   (c) $A < B$ (+1) does not make sense

   (+1) [A and B are points with no 'greater than' relationship

   (d) $A \in \overline{CD}$ (+1) makes sense

   (+1) [$\overline{CD}$ is a line segment which contains lots of points.
   A is a point & may lie in the line segment.

   (e) $AB \in \overline{CD}$ (+1) does not make sense

   (+1) [AB is a distance between A and B.
   $\overline{CD}$ is comprised of points not distances

2. Consider: $\forall n \in \mathbb{R}$, $\frac{d}{dx}x^n = n \cdot x^{n-1}$.

   (a) [2] (ProofsActivity #3) Use as few symbols as possible to interpret the meaning with words. (+1)

   For all real numbers n, we have the derivative of the function x^n (with respect to x) is the function $n \cdot x^{n-1}$. (+1)

   (b) [2] (HW1 #1.13) Negate the above statement symbolically without using the $\forall$ symbol. $\neg(\forall n \in \mathbb{R}, \frac{d}{dx} x^n = n \cdot x^{n-1})$ is equivalent to

   $\exists n \in \mathbb{R} \ni \frac{d}{dx}(x^n) \neq n \cdot x^{n-1}$ by de morgan's law.

not (.5)
DeMorgan (.5)
correct demorgan (.5)
Symbols (.5)

3. [2] (Complete this question last!!) What concept did you study well and not see on the exam?

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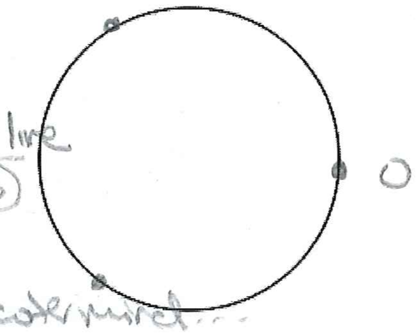
Definition 1. *If A and B are arbitrary points with coordinates a and ~~B~~ b respectively, then the* distance AB *from A to B is defined as:*

$$AB = \begin{cases} |a-b| & \text{if } |a-b| \le \lambda \\ 2\lambda - |a-b| & \text{if } |a-b| > \lambda \end{cases}$$

4. Consider the circle with circumference 2π so we let $\lambda = \pi$ in the distance defined above.

   (a) [2] (Practice3.1) Find the distance between the points with coordinates $\frac{\pi}{4}$ and $\frac{9\pi}{4}$.

   $|9\pi/4 - \pi/4| = 8\pi/4 = 2\pi > \pi$ so use 2nd line (+.5)

   (+1)

   $2\pi - (2\pi) = 0$ (+.5)

   makes sense since $\pi/4$ and $9\pi/4$ are coterminal...

   three points (+1)

   they work (+2)

   (b) [3] (1dFolding Activity #5) Recall on the line that B is *between* A and C if $AB + BC = AC$. Let us define *between* similarly on the circle. Identify three points on the circle such that no point is between the others.

   Let A have coord 0, B have coord $2\pi/3$ and C have coord $-2\pi/3$

   note i did not ask them to prove it?

   Then $AB = |0 - 2\pi/3| = 2\pi/3$ and $AC = |0 - -2\pi/3| = 2\pi/3$ so

   $BC = 2\pi - |2\pi/3 - -2\pi/3| = 2\pi/3$ no combo of A-B-C, A-C-B or C-A-B will be satisfied.

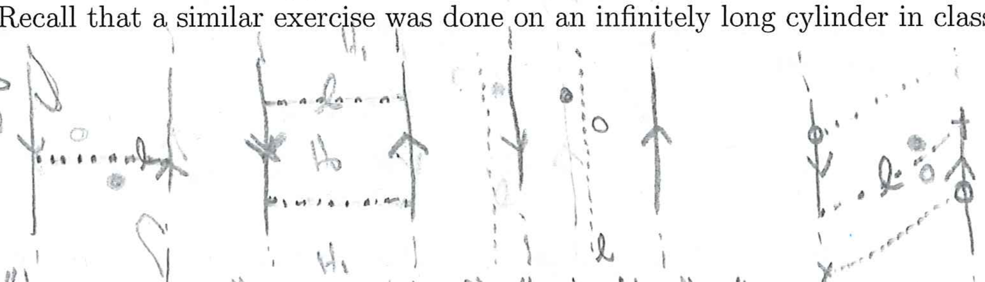
5. (Activity 4.5) A Möbius strip can be considered as a vertically infinity strip of paper with left and right sides glued together via a half-twist.

   (a) [4] There are several possible "lines" that might be defined on this space. Describe (and consider sketching!) at least three significantly different shaped "lines". (Recall that a similar exercise was done on an infinitely long cylinder in class.)

   Start (+1)

   (+1.5) using infinite Mobius

   (+1.5) each line type

   "horizontal line" "working lines" "vertical line" "weird helix"

   (b) [3] Identify which of the kinds of "lines" you found above satisfy Postulate 3. (Every line l determines a decompositions of the plane into 3 distinct sets: H_0, H_1, and l where:

   - Every pair of points in one of the H_i are on the same side of l, and
   - Every pair of points where one is in H_0 and the other is in $H_!$ are on opposite sides of l.)

   reasoning +1

   remove 1 as possible (+1)

   got one that worked (+1)

   did not list ones that didnt (+1)

   ~~"horizontal line"~~
   ~~"vertical line"~~
   ~~"weird line"~~
   note that the heart is attached above the empty & solid dot are on the same side but the line connecting them would cross l

   "working lines" or lines that loop back on themselves work?

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6. [5] (2Dintro Activity) Critique the following theorem and proof.

logic error (+2)
make comment (+1)
sense/true (+2)

Theorem 1. *Two distinct lines can intersect in at most one point*

Proof. Pick distinct points A, B, and C. Then Postulate 1 implies that both $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ determine unique lines, and then intersect in point A, so there is one point of intersection. $\square$

The theorem assumes two distinct lines but the proof starts with a different set of assumptions. In fact the proof's assumptions don't even follow from the Theorem's assumption so the proof will not be valid.

7. [8] (HW2 #2.29/2.7) Prove the following: Let A and B be points with coordinates a and b respectively, and let ϕ be the fold with crease B. Then the coordinate of $\phi(A)$ is $2b - a$.

start (+1)
logic (+3) use thms +1, all cases +1.5, overall 1.5
notation (+2)
structure/intro (+2)

We will consider three cases depending on the relation between a & b.

Case 1: If $a=b$ then $A=B$ which means that A does not move.
Note $2b-a = 2b-b = b$ since $a=b$
The coord. of A remained the same. ✓

Case 2: If $a<b$, then $AB = b-a$.
Since ϕ is a fold distance is maintained
so $AB = \phi(A)\phi(B) = \phi(A)B$.
B is between A and $\phi(A)$ and since $a<b$
we know the coord of $\phi(A) > b$.
Thus the coord of $\phi(A)$ is $b + \phi(A)B = b + (b-a)$
or $2b-a$.

Case 3: If $b<a$, then $AB = a-b$.
Since ϕ is a fold, we still know distance is maintained
so $AB = \phi(A)\phi(B) = \phi(A)B$
B is between A and $\phi(A)$ and since $b<a$
we know the coord of $\phi(A) < b$
Thus the coord of $\phi(A)$ is $b-(a-b) = 2b-a$. //

This concludes all possible cases.

note: many let $\phi(A)$ have coord x and used $AB + \phi(A)B = \phi(A)A$ to solve for x which is nice.

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Like

Postulate Origametry 1. *Any two points determine a unique line.*

stronger

Postulate Origametry 2. *Given any two distinct points on a line, there is a one-to-one correspondence, called a* ruler *between all points on the line with the real numbers that sends one of the two given points to 0, and the other to some number greater than 0. The number p assigned to a point P by the ruler is called the* coordinate.

Postulate Euclid 1. *We can draw a unique line from any point to any point.*

Postulate Euclid 2. *We can increase a finite line continuously into a line.*

suggest coord while implied

8. [9] The first two postulates for 2 dimensions from our textbook are above along with the first two postulates created by Euclid about 2219 years ago to describe geometry. Recall that postulates are "common notions" which "everyone" agrees must be true and so can be assumed and used to build theorems. Generally this question is asking you to compare and contrast these different postulates chosen by the author of our text and Euclid. Does one set imply the other and/or vice versa? Does one set seem superior to you than the other and if so explain why. (You do not have to prove anything formally in this exercise, but you do have to provide compelling evidence.)

Sted (+1)

Does one set imply the other & justification (+3)
Does one seem superior to you & explain (+3)
Compare & contrast (+2)

if make false claim -1 for each

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