

Sets, Graphs, & Invariants

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Let $S = \{a, b, c\}$ and $T = \{a, b\}$. We want to show $S \not\cong T$.

(a) If we use the definition of \cong , what would we need to prove?

(b) Find another tool/theorem/approach that would be easier to show $S \not\cong T$?

2. Brainstorm (or scan Giusti's text) tools/theorems/approaches that we could leverage to show two graphs are *not* isomorphic. Note several definitions are given below.

(a) Try to leverage the idea of the *degree*.

(b) Try to leverage the idea of the *degree sequence*.

(c) Try to leverage the idea of graph homomorphisms from line graphs of length n .

(d) Try to leverage the idea of path components.

Definition 0.1. Let $\Gamma = (V, E)$ be a graph with $v_i \in V$. The *degree* of v_i is the number of edges that contain v_i . The *degree sequence* of Γ is the list of degrees of all vertices in V in descending order.

Definition 0.2. The *line graph of length 1*, is the graph $L = (\{v_1, v_2\}, \{v_1v_2\})$.

The *line graph of length 2*, is the graph $L = (\{v_1, v_2, v_3\}, \{v_1v_2, v_2v_3\})$.

The *line graph of length 3*, is the graph $L = (\{v_1, v_2, v_3, v_4\}, \{v_1v_2, v_2v_3, v_3v_4\})$.

The *line graph of length n* , is the graph $L = (\{v_1, \dots, v_{n+1}\}, \{v_1v_2, v_2v_3, \dots, v_nv_{n+1}\})$.

