## Sets, Graphs, & Invariants

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.
- 1. Let  $S = \{a, b, c\}$  and  $T = \{a, b\}$ . We want to show  $S \not\cong T$ .
  - (a) If we use the definition of  $\cong$ , what would we need to prove?
  - (b) Find another tool/theorem/approach that would be easier to show  $S \not\cong T$ ?
- 2. Brainstorm (or scan Giusti's text) tools/theorems/approaches that we could leverage to show two graphs are *not* isomorphic. Note several definitions are given below.
  - (a) Try to leverage the idea of the *degree*.
  - (b) Try to leverage the idea of the *degree sequence*.
  - (c) Try to leverage the idea of graph homomorphisms from line graphs of length n.
  - (d) Try to leverage the idea of path components.

**Definition 0.1.** Let  $\Gamma = (V, E)$  be a graph with  $v_i \in V$ . The *degree* of  $v_i$  is the number of edges that contain  $v_i$ . The *degree sequence* of  $\Gamma$  is the list of degrees of all vertices in V in descending order.

**Definition 0.2.** The line graph of length 1, is the graph  $L = (\{v_1, v_2\}, \{v_1v_2\})$ . The line graph of length 2, is the graph  $L = (\{v_1, v_2, v_3\}, \{v_1v_2, v_2v_3\})$ . The line graph of length 3, is the graph  $L = (\{v_1, v_2, v_3, v_4\}, \{v_1v_2, v_2v_3, v_3v_4\})$ . The line graph of length n, is the graph  $L = (\{v_1, ..., v_{n+1}\}, \{v_1v_2, v_2v_3, ..., v_nv_{n+1}\})$ . Recall the Math Major's fifth listed Student Learning Objective (SLO): "Math majors will be able to describe how mathematical or quantitatively-based arguments affect society"

1. Let's take a snap shot of where we are now... how would you "describe how mathematical or quantitatively-based arguments affect society"?

2. Can a project in this class better inform your answer to the above? If so, how? If not, what academic assignment would better inform your answer to the above?