Homotopy

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

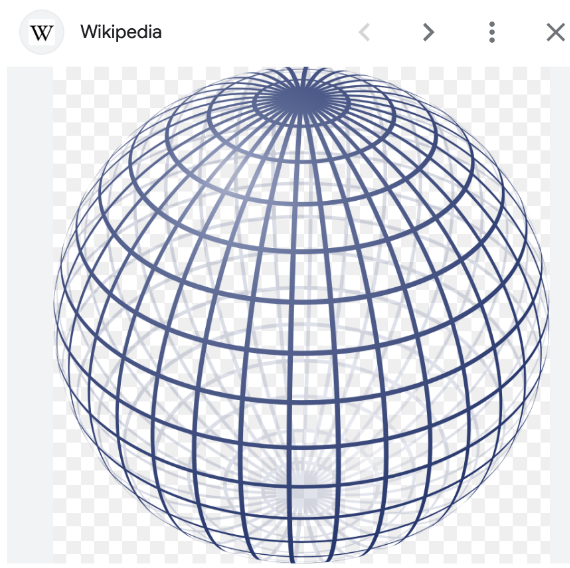
Definition 0.1. Let X and Y be topological spaces. A *homeomorphism* $h : X \to Y$ is a continuous function with a continuous two-sided inverse. If such a map exists, X and Y are *homeomorphic* and is denoted $X \cong Y$.

Definition 0.2. Let X and Y be topological spaces and f, $g : X \to Y$ be continuous maps. We say f is *homtopic* to g, denoted $f \simeq g$, if there is a continuous function $H : I \times X \to Y$ with $H(0, x) = f(x)$ and $H(1, x) = g(x)$. We call H the homotopy between f and g.

1. Let $e : S^1 \to S^2$. Recall that $S^1 = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 = 1\}$ and $S^2 = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = 1\}$. Define $e(x, y) = (x, y, 0)$. Define also $p : S^1 \to S^2$ by $p(x, y) = (0, 0, 1)$.

   (a) Sketch the images of e and p.

   (b) Is $e \simeq p$? If so, describe the homotopy. (You do *not* need to write down an algebraic rule for this.)



Definition 0.3. Let X and Y be topological spaces, $f : X \to Y$ and $g : Y \to X$ be continuous maps. If $g \circ f \simeq \mathrm{id}_X$ and $f \circ g \simeq id_Y$, we say that X and Y are *homotopy equivalent* and write $X \simeq Y$. If a space is homotopy equivalent to a point, it is *contractible*.