TMATH 342: WrittenHW 4

Read Giusti's "Thinking Topologically" pages 23-26 and then do:

1. Let $f : S^1 \to \mathbb{R}^3 \setminus \{0\}$ defined by $f(x, y) = (x, y, 0)$.
   (a) [2] Draw the image of f in $\mathbb{R}^3 \setminus \{0\}$.
   (b) [3] Show f is null-homotopic. If you do not write down the homotopy explicitly, provide illustrations and/or descriptions of the homotopy.
2. Let $S^1 \vee S^1 = \{(x, y) \in \mathbb{R}^2 | (x-1)^2 + y^2 = 1 \text{ or } (x+1)^2 + y^2 = 1\}$.
   Let $X = \mathbb{R}^2 \setminus \{(-1, 0), (1, 0)\}$.
   (a) [2] Draw $S^1 \vee S^1$ in $\mathbb{R}^2$.
   (b) [2] Draw/shade X in $\mathbb{R}^2$.
   (c) [3] Use an illustration and/or description to demonstrate $S^1 \vee S^1 \simeq X$.
3. [3] Define a topological space X, that is homotopic to S^2 but not homeomorphic to S^2. Provide intuition (not justification) for your answer.

Note, in this class, it is more important to communicate clearly, than it is to be correct! Make sure that you edit your work (and your peers!!) so that your completed homework is easily understood!