Key

Midterm TMath 342 /45 Spring 2019

Show all your work. Reasonable supporting work must be shown to earn credit.

1. [9] TRUE or FALSE: If the statement is *always* true, mark it as true, otherwise, mark the statement as false. Either way give a brief explanation for your answer.

   (a) (HW1 9.11) There is a knot with a crossing number of one.

   False. Given one crossing there are only 3 ways of connecting the ends (up to rotation). They are:

   Cannot connect remaining ends.

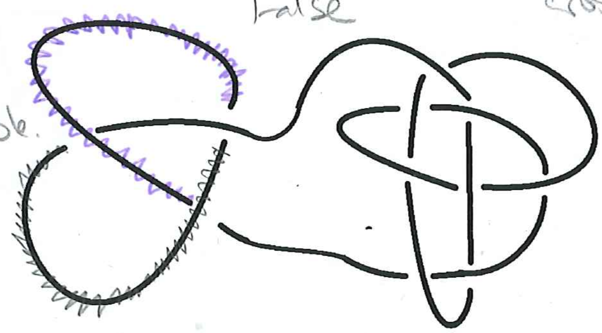
   The results are either the unknot (w/ crossing # zero) or not a knot. Thus ∄ knots w/ crossing # of 1.

   start (+.5)
   false (+.5)
   logic (+1)
   all cases (+.5)
   sense/true (+.5) def. (+.5)

   (b) (HW2 9.7) The knot $3_1\#8_{17}$ (shown below) is equivalent to the unknot.

   False

   Notice $3_1\#8_{17}$ is tricolorable. The unknot is not tricolorable. Tricolorability is a knot invariant thus $3_1\#8_{17}$ is not equivalent to the unknot.

   start (+.5)
   false (+.5)
   logic (+1)
   sense/true (+1)

   (c) (ChemicalSymmetryActivity) A ~~chemical~~ molecule that is chemically chiral is necessarily geometrically chiral.

   True

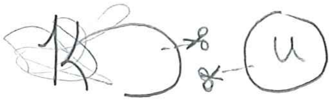
   start (+.5)
   true (+.5)
   logic/reasoning (+1)
   definitions (+1)

   A molecule that is chemically chiral cannot be deformed into its mirror image - even with the extra flexability inherent in the molecular structure (ex propellors). Geometrically we have less movements available to us so if we could not show the molecule was equivalent to its mirror before - when we had the extra flexability - we certainly wouldn't be able to do it when constrained by rigid motions/geometrically.

2. [3] (ConnectSum Activity #5) Find a knot U such that for any knot K, that $K\#U$ is equivalent to K. Justify yourself.

start (+.5)
unknot (+1)
Connect Sum def (+.5)
logic/sense (+1)

The unknot will work. Denote the unknot as U.
Let K be a knot. We walk thru the steps of connect sum for K#U.
Choose an arc for K and U and make a cut.



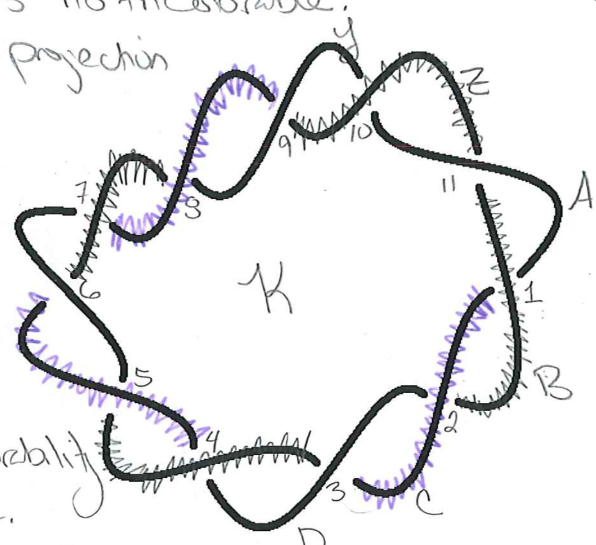
Notice that once U is cut we have a simple arc.

Thus when we connect the ends of K's cut to the ends of U's cut we essentially 'patch' the cut in K.

3. [6] (Invariants Activity #2) Determine if the knot whose projection is below is tricolorable. If it is, tricolor it. If not, justify yourself.

start (+1)
tricolorable def (+1)
proof intro/style (+2)
logic (+2)

We will show the knot K is no tricolorable.
Let A and B be arcs in the projection labeled to the right. We consider 2 cases:



Case 1: Assume A and B are the same color, WLOG let A and B be black.
Since crossing #1 has 2 arcs that are black, tricolorability forces arc C to be black.
Similarly crossing #2 forces D to be black.
The argument continues thru each crossing forcing all of K to be black ⇒ A and B cannot be the same color.

Case 2: Assume A and B are different colors (shown above).
Crossing 1 + tricolorability forces C to be the 3rd color (purple)
Similarly crossing 2 + tricolorability forces D to be the 1st color (black).
This argument systematically works thru each crossing.
Note crossing 9 + tricolorability forces arc Z to be grey and crossing 10 + tricolorability forces arc A to be purple.
But arc A was black. ⊠.

4. [4] (12.7) An object can fail to be a Topological Rubber Glove in multiple ways. Describe two ways that an object can fail to be a Topological Rubber Glove.

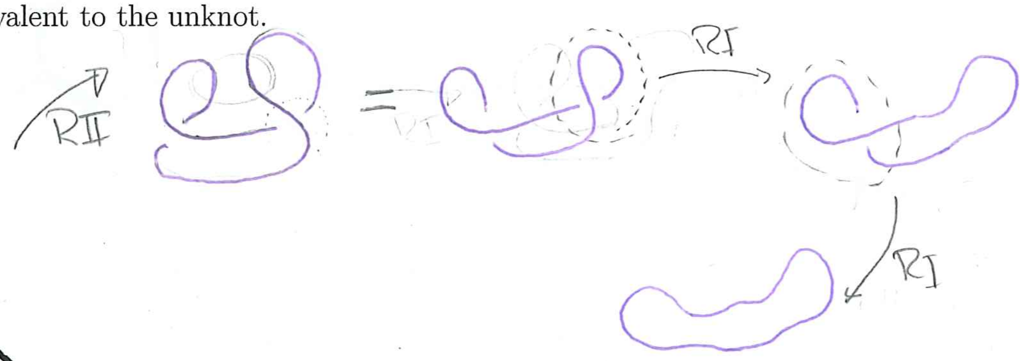
start (+.5)
Def of TRG (+1.5)
how can fail (+2)

Recall a Topological Rubber Glove is an object that
1) It can be deformed into its mirror image and
2) There is no configuration (assuming complete flexibility) that is geometrically achiral.

So either 1 can fail (ex The trefoil cannot be defored into its mirror) or 2 can fail (ex figure 8 can deform to its mirror image but has position that is geom. achiral)

5. [4] (HW1 9.9) Restrict yourself to one Reidemeister move at a time to show the knot below is equivalent to the unknot.

start (+.5)
use Reidemeister Moves (+2)
only (+1)
sense/follow (+.5)



which is the unknot.

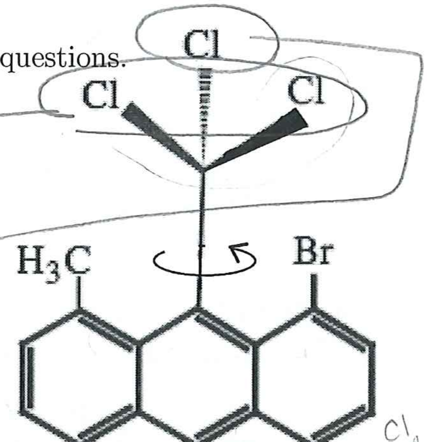
6. [4] (Lecture 4/11) Explain how the Reidemeister moves are relevant/powerful beyond showing equivalency between two given knots.

start (+.5)
beyond equivalence (+1.0)
connect to invariants (+1.0)
sense/follow (+.5)
true (+1)

Reidemeister moves can be used to prove invariants are invariants - that is that the property described (such as tricolorability) is not dependent on the projection of the knot.

"Reidemeister moves preserve tricolorability"

7. Consider the molecule on the right for the following questions.



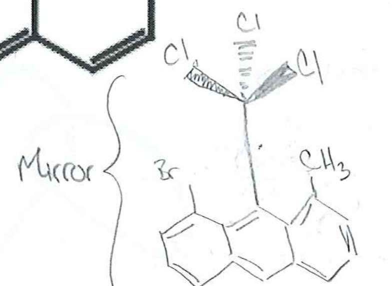
(a) [1] (ChemMirror Activity #4) Indicate what sections (if any) of the molecule are in front of the paper,

(b) [1] (ChemMirror Activity #4) Indicate what sections (if any) of the molecule are in back of the paper,

(c) [4] (12.2) Determine if the molecule is geometrically chiral in its current configuration. Explain your reasoning.

start (+.5)
Geom chiral def (+1) (mid)
got it (+.5)
reasoning/process (+1.5)
sense (+.5)

Geometrically chiral
Notice to get the 3 hexagonal rings w/ Br & H_3C to match the mirror we'd like to flip the molecule over the dotted line.
However the 'propellor' won't be oriented correctly then, 2 of the propellors would be behind the paper.



(d) [4] (12.2) Determine if the molecule is chemically chiral or not. Explain your reasoning.

start (+.5)
Chem chiral def (+.5)
got it (+.5)
reasoning/process (+1.5)
sense (+.5)

Chemically achiral
We can flip the molecule across the dotted line & then spin the propellor to get the Cl's in the correct orientation.

(e) [2] (12.2) Determine if the molecule is topologically chiral or not. Explain your reasoning.

start (+.5)
top chiral def (+.5)
got it (+.5)
reasoning (+.5)

Topologically achiral
If we can get the molecule to look like it's mirror with a lot of chemically induced flexability - then we can certainly get it to look like its mirror with more flexability

(f) [3] Determine if the molecule is a Euclidean Rubber Glove or not. Explain your reasoning.

start (+.5)
ERG def (+1)
sense (+.5)

Not a Euclidean Rubber Glove.
Note: the chemical is chemically achiral (1st condition)

(+1) [but we can move the propellor to be sym. w/respect to the 3 hexagonal rings. Shown →
This configuration is geometrically achiral