Worksheet: Key

1) a) $F(3-4i) = \begin{bmatrix} 3 & -(-4) \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$

b) $F(1) = F(1+0i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $F(i) = F(0+1i) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2) The function F is one-to-one if $a+bi \neq \alpha+\beta i$
$\Rightarrow F(a+bi) \neq F(\alpha+\beta i)$

Note $F(a+bi) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and $F(\alpha+\beta i) = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}$

Since $a+bi \neq \alpha+\beta i$ either $a \neq \alpha$ or $b \neq \beta$
thus the 2 matrices $F(a+bi) \neq F(\alpha+\beta i)$
are not equal.

3) Let $z = a+bi$ and $w = \alpha+\beta i$
Then
$F(z)+F(w) = F(a+bi)+F(\alpha+\beta i) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} + \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}$

$= \begin{bmatrix} a+\alpha & -(b+\beta) \\ b+\beta & a+\alpha \end{bmatrix} = F(a+\alpha+i(b+\beta))$

$= F((a+ib)+(\alpha+i\beta)) = F(z+w)$ //

4) $[F(i)]^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$= F(-1) = F(i^2)$ //

5) Let $z = a+ib$ and $w = \alpha+i\beta$.

$$F(zw) = F((a+ib)(\alpha+i\beta)) = F(a\alpha - b\beta + i(a\beta + b\alpha))$$

$$= \begin{bmatrix} a\alpha - b\beta & -(a\beta + b\alpha) \\ a\beta + b\alpha & a\alpha - b\beta \end{bmatrix}$$

$$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} = F(a+ib) \cdot F(\alpha+i\beta)$$

$$= F(z)F(w)$$