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Math 341: Quiz 6

Name: KEY

True/False: If the statement is true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [3] If $A = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 4 & -2 & 1 & 3 \end{bmatrix}$ and $T_A : \mathbb{R}^4 \to \mathbb{R}^2$ is the corresponding linear operator, then $[1\ 2\ 0\ 0]^\top$ is an element of the kernel of T_A.

$$T_A\left(\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 4 & -2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ so true

2. [3] If $T : V \to W$ is a linear operator that is onto, then T is invertible.

false: Let $T : \mathbb{R}^2 \to \mathbb{R}$ note T is onto
$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto x = [1\ 0]\begin{bmatrix} x \\ y \end{bmatrix}$

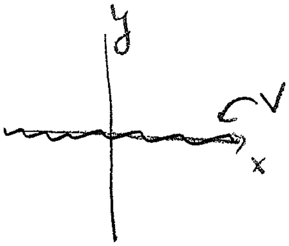
but T is not invertible b/c it is not one-to-one.

3. [3] Let $S = \{\vec{v_1}, ...\vec{v_k}\} \subseteq V$, where V is a vector space of dimension n. If S spans V then $k \leq n$.

false. Let $V = \mathbb{R}^2$
and $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.
note S spans V but $k = 3 \not\leq 2 = \dim V$

4. [3] If U and V are subspaces of finite-dimensional vector space W, then $\dim(U + V) = \dim U + \dim V$.

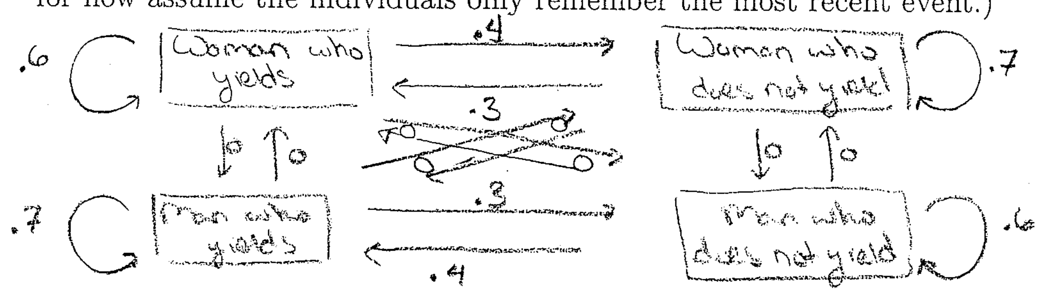
false Let $U = sp\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and $V = sp\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$



$U = \mathbb{R}^2$
$W = \mathbb{R}^2$ note: The whole space is a subspace of itself.
$U + V = \mathbb{R}^2$
$\dim(U+V) = 2 \neq \dim(U) + \dim(V) = 2 + 1$

5. H. Peyton Young provides "The Etiquette Game" from the book *Individual Strategy and Social Structure*. Two people of opposite genders approach a doorway and have the option to either yield to let the other person pass through, or not yield. Their interaction will affect future behavior in a similar meeting situation. The four states and the probabilities of individuals changing states are given by the graph below.

(It should be noted that this system assumes the individuals involved only remember the previous encounter. More memory can be brought into the Markov process, but for now assume the individuals only remember the most recent event.)



(a) [4] Define the following:

- w_1 is the probability of a person being both a woman and choosing to yield.
- x_1 is the probability of a person being a woman who chooses not to yield.
- y_1 is the probability of a person being both a man and choosing to yield.
- z_1 is the probability of a person being a man who chooses not to yield.

Find the equation, in closed form, that will allow you to compute the probabilities of each state after 10 rounds of the etiquette game. Do *not* actually carry out the computations.

To find $\begin{bmatrix} w_2 \\ x_2 \\ y_2 \\ z_2 \end{bmatrix}$ I'd use $\begin{bmatrix} .6 & .3 & 0 & 0 \\ .4 & .7 & 0 & 0 \\ 0 & 0 & .7 & .4 \\ 0 & 0 & .3 & .6 \end{bmatrix} \begin{bmatrix} w_1 \\ x_1 \\ y_1 \\ z_1 \end{bmatrix}$, so to find after 10 games: $\begin{bmatrix} .6 & .3 & 0 & 0 \\ .4 & .7 & 0 & 0 \\ 0 & 0 & .7 & .4 \\ 0 & 0 & .3 & .6 \end{bmatrix}^{10} \begin{bmatrix} w_1 \\ x_1 \\ y_1 \\ z_1 \end{bmatrix}$

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(b) [4] Let A represent the transition matrix you set up in (a). Find a basis for $\mathcal{N}(I_4 - A)$.

$$I_4 - A = \begin{bmatrix} .4 & -.3 & 0 & 0 \\ -.4 & .3 & 0 & 0 \\ 0 & 0 & .3 & -.4 \\ 0 & 0 & -.3 & .4 \end{bmatrix} \rightarrow \begin{bmatrix} 4/10 & -3/10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3/10 & -4/10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Solset} = \{(\tfrac{3}{4}x, x, \tfrac{4}{3}y, y) \mid x, y \in \mathbb{R}\} = \left\{ \begin{bmatrix} 3/4 \\ 1 \\ 0 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4/3 \\ 1 \end{bmatrix} y \;\middle|\; x, y \in \mathbb{R} \right\}$$

$$= \text{span} \left\{ \begin{bmatrix} 3/4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4/3 \\ 1 \end{bmatrix} \right\}$$

$$\text{basis}: \left\{ \begin{bmatrix} 3/4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4/3 \\ 1 \end{bmatrix} \right\}$$