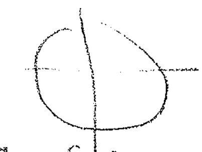
Math 341: Quiz 4

Name: KEY

True/False: If the statement is true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [3] The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T([x\ y]^T) = [\cos x\ \ \sin y]^T$ is linear.



False $\quad T\begin{bmatrix}\pi\\ \pi\end{bmatrix} = \begin{bmatrix}\cos\pi\\ \sin\pi\end{bmatrix} = \begin{bmatrix}-1\\ 0\end{bmatrix}$

$T\begin{bmatrix}\pi/2\\ \pi/2\end{bmatrix} + T\begin{bmatrix}\pi/2\\ \pi/2\end{bmatrix} = \begin{bmatrix}\cos\pi/2\\ \sin\pi/2\end{bmatrix} + \begin{bmatrix}\cos\pi/2\\ \sin\pi/2\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ 2\end{bmatrix}$

but $\begin{bmatrix}-1\\ 0\end{bmatrix} \neq \begin{bmatrix}0\\ 2\end{bmatrix}$

2. [3] In an abstract vector space, $c\vec{0} = \vec{0}$ for any scalar c.

true. Since $\vec{0} = \vec{0} + \vec{0}$

$c\vec{0} = c(\vec{0} + \vec{0}) = c\vec{0} + c\vec{0}$ by dist.

Note that $-c\vec{0} \in V$ so if we add $-c\vec{0}$ to both sides of the above we'll have

$\vec{0} = c\vec{0}$. //

3. [3] Let $\vec{v_1}, \vec{v_2} \in \mathcal{P}_2$ where $\mathcal{P}_2$ is the vector space of all polynomials with degree less than or equal to 2. If $\vec{v_1} \neq \vec{v_2}$, then $\text{span}\{\vec{v_1}\} \neq \text{span}\{\vec{v_2}\}$.

false $\quad$ let $\vec{v}_1 = x$ and $\vec{v}_2 = 2x$.

note $\vec{v}_1 \neq \vec{v}_2$

but $\text{span}\{\vec{v}_1\} = \text{span}\{\vec{v}_2\}$.

4. [3] If $U = \text{span}\{\vec{v_1}, \vec{v_2}\}$ and $W = \text{span}\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$, where $\vec{v_1}, \vec{v_2}, \vec{v_3}$, and $\vec{v_4}$ are all from some vector space V, then W is strictly larger then U.

false. $\quad$ Let $\vec{v}_1 = \vec{v}_2 = \vec{v}_3 = \vec{v}_4 = \vec{0}$.

then $U = \{\vec{0}\} = W$

so W is <u>not</u> strictly larger than U

Free Response: Show all your work and justify your steps. No credit is given for the correct answer with no justification.

5. Let $V = \{\vec{T} : \mathbb{R}^3 \to \mathbb{R}^2 | \vec{T} \text{ is a linear function}\}$. The elements in our vector field are thus *linear maps*. Define vector addition and scalar multiplication by the following:

$$(\vec{T}+\vec{R})\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) + \vec{R}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \qquad (\alpha\vec{T})\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \alpha\left(\vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)\right)$$

where $\alpha \in \mathbb{R}$, and $\vec{T}$, $\vec{R} \in V$.

It is given that V forms a vector space under the defined operations above. In particular note that the zero vector is the linear map defined by $\vec{0}([x\ y\ z]^T) = [0\ 0]^T$ for all $[x\ y\ z]^T \in \mathbb{R}^3$.

(a) [2] Given a subset W of V, what properties must you exhibit to verify that W is a subspace of V?

(1) The zero vector of V is also contained in W
(2) Given $\vec{v}, \vec{u} \in W$, then $\vec{v}+\vec{u} \in W$
(3) Given $\alpha \in \mathbb{F}$ & $\vec{v} \in W$ then $\alpha\vec{v} \in W$

(b) [6] Show that

$$S = \{\vec{T} \in V | \exists\, A \in \text{Mat}_{2x3}(\mathbb{R}) \text{ so that } \vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \text{ for all } \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \in \mathbb{R}^3\}$$

forms a subspace of V.

stated (+1) got it (+1)

(1) Note $\vec{0} : \mathbb{R}^3 \to \mathbb{R}^2$ defined above can be written

$\vec{0}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$ thus $\vec{0} \in S$.

stated got it (+1)

(2) Let $T_A, T_B \in S$ be the linear functions corr to the matrices $A+B$ acting on the left resp.
Then $T_A + T_B$ has the matrix $[a_{ij} + b_{ij}]$ assoc with it
So $T_A + T_B \in S$.
Explicitly $(T_A+T_B)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

stated got it (+1)

(3) Given $\alpha \in \mathbb{R}$ & $T_A \in S$. The matrix assoc. with the linear funct αT_A is $\begin{bmatrix} \alpha a_{11} & \alpha a_{12} & \alpha a_{13} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} \end{bmatrix}$. So $\alpha T_A \in S$.